A STUDY OF STARS OF EARLY SPECTRAL TYPE

with special reference to emission line spectra.

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PART I.

Emission Lines in Early Type Spectra.

General Survey.

Abstract.

The present state of the study of emission line stars of early spectral types is briefly reviewed, mainly from the observational point of view.

A rational theory of stellar spectra connects the spectrum of a star with the fundamental stellar characteristics such as effective temperature, surface gravity and atmospheric constitution. Three steps towards such a theory are recognised. The first is observation with the necessary developments of instruments and methods; second is classification of observation in such a way as to bring together all spectra having the same (or nearly the same) features. The last, and perhaps the most difficult, step is interpretation. With the help of "justifiable" experimental and theoretical considerations, the physical and chemical properties of the source of the spectrum are inferred. These three steps are not independent; they merge into each other.

Observation of Be-Stars.

For example, consider the group of early-type stars which have bright lines in their spectra. For these stars, observations began some seventy-five years ago, in 1866, when Secchi saw bright "Balmer" lines in γ Cas. and β Lyr. The number of these stars stands now well over 400, only five of which were discovered visually. The objective prism survey at Harvard and the Hα - survey at Mt. Wilson account for the majority of the rest (1). Stars fainter than the eighth magnitude are not yet covered by observations, and the present number of emission line stars represents mainly stars brighter than that limit. A new method for detecting objects having emission spectra has been recently suggested by Seyfert (2). The method consists of photographing stars through a red filter at two different focal settings, one corresponding to Hα -wave-length, and the other corresponding to a shorter wave-length, chosen in such a way that the two images for a normal star are of the same size. For an object with a strong Hα, the first image will be stronger. The method is promising, but so far it is limited to exceptional Be-stars (3). The definition of a bright line star has also been modified. Previously, a star was described thus, if it has one or more lines brighter than the continuous background; but many stars have absorption lines, either so weak or of such a peculiar contour, as to leave no doubt about the presence of incipient emission. Now these stars are considered to belong to the emission line group. There is no rigid rule about this definition, which may be extended to cover many stars, since as a rule there must be some emission in the centre of the line. This question is bound to be of importance, as more spectro-photometric work on Be-stars is done. Quoting Merrill (4) still :-

> " ..... in the present state of stellar spectroscopy little attention is likely to be paid to a bright line spectrum unless at least one line is conspicuous."

One has to wait some advance in "the present state of spectroscopy", but one may wonder how far the incompleteness of the observation is affecting one's conclusions, especially statistical results, about emission line stars.

Classification of Be-Stars.

As regards classification (5), it was begun also by Secchi, who was concerned with the appearance of the spectrum.

He classified all his observations in four types, but γ Cas. and β Lyr. were unique, so he placed them separately in his fifth type. In subsequent systems of classification, an eye was kept on the development and evolution of spectra, beside their formal character. This led Vogel (1874) and Lockyer to place emission line objects together in one class. On the other hand the Harvard observers placed early bright line stars nearer normal stars; thus in the Pickering-Fleming classification they were given the separate class D , and described as stars of Orion type with bright lines by Miss Maury. In the finally accepted system, bright line stars were kept in sequence parallel to the normal stellar sequence, and a suffix is added to denote their special character. Miss Cannon used the suffix "p" for all abnormalities in the spectrum, later the letter "e" was adopted for emission. Merrill (6) classified Be-stars, according to the strength of the bright lines, in four groups :-

(1) γ Cass. group with intense bright $H\alpha$ & strong bright $H\beta$

(2) b^2 Cyg. " " " " " " " reversed $H\beta$

(3) Electra group with weak bright $H\alpha$ & absorption $H\beta$

(4) ϕ Per. group with bright strong $H\alpha$, bright reversed $H\beta$ and emission variable. To these another group amy be added -

(5) P Cygni with strong emission and shifted absorption.

In later years many distinctions were found between narrow and wide-line stars, and therefore in some discussions, Be-stars are divided into Bne (n for nebulous line) and Bse (s for sharp). The presence of metallic lines in emission is usually indicated by a suffix (m) or Fe for iron lines; in spectra where Balmer lines are bright, the last member is given , thus the full "shorthand code" of Cas. is B o m n e θ , where H_θ is the last bright Balmer line. The classification of Be-stars is connected with the next step of physical interpretation. But there is always the danger of classifying stars according to a preconceived opinion. A desirable feature of a system of classification is (7) that, "It must be based exclusively upon observable characteristics of the Spectra. Characteristics clearly recognisable in a large number of spectra, but not yet, or not fully interpreted physically, are more valuable criteria than difficultly observable details which are supposed to be well understood." From this point of view the Draper system is almost as perfect as possible except at the ends of the sequence where modifications were necessary. For early stars, the extension to O stars by Plaskett (8), and to W - R stars by Beals (9) fit well with the above specifications. Thus the Draper system will be used in the present study.

Interpretation of Be-stars.

As regards physical interpretation, Be-stars represent a two-fold problem. First, the absorption line spectrum has to be explained; this is a problem concerning all B-stars, and has its place in general theories of stellar evolution and constitution. Second, the emission line spectrum has to be discussed. The main point here is to provide a theory explaining the excitation of such lines, which is similar to the case of nebulae and novae, within the frame of a B-type atmosphere.

A very concise summary of the present state of emission line spectra was given by H. N. Russell (10); there seems to be nothing better than restating some of his conclusions.

"I: The presence of emission lines (even of unknown origin) indicates some important facts about the conditions under which they originate.
1. Emission lines appear only under conditions widely different from thermodynamic equilibrium. 'Absorption' lines indeed, depend also upon such departures - there could be none in a 'Holmraum' - but emission lines in general demand more extensive deviations.
2. Such lines must orginate in a gas of very low, at least fairly low density, where collisions are rare.
3. It follows that the energy of these radiations is not autogenous (originating within the emitting gas) but is fed into the gas in some form and transformed by the emitting atoms.
4. In many cases it is evident (and in all cases probable) that they arise in an envelope of low density external to the original source of energy, sometimes adjacent to it and sometimes very remote.
....
8. Emission lines (in the observable region between 8000 and 3000A) are very rare in the spectra of stars in which the maximum energy of the continuous background falls in this region (classes A to K) but are much more common when this maximum is in the infra-red (class M) in the ultra-violet (O, B).

II: Emission lines can obviously best be studied in the cases where the emitting gas can be observed separately from the exciting source. We then find evidence of excitation of at least four different kinds; resonance, fluorescence, collisions and recombinations ....

III. Emission lines in stars which appear telescopically single suggest the existence of envelopes too close to be directly resolved. There are several distinct classes of these objects.

(a) Hot stars with emission lines. Here we may set up an apparently almost continuous sequence from planetary nebulae and novae, where the existence of emitting envelope is certain, through Wolf - Rayet stars where it appears extremely probable, to ordinary Be-stars. (It is not suggested that there is a continuous sequence of physical processes involved in the production of the envelopes!)

In normal Be-stars only a few lines are bright - Often only those of hydrogen - sometimes also He and FeII etc. A noteworthy property of these spectra is their variability. The emission often vanishes, or appears, or shows fluctuations with intervals of a number of years, but not exactly periodic. Evidently the condition which gives rise to emission depends on a rather delicate balance of conditions which is easily disturbed. For these stars it is always permissible to assume an extensive envelope of low density and plenty of ultra-violet light.

The Hα line is always the brightest and the ultra violet hydrogen lines are almost always dark. This is a natural consequence of the gradient of intensity, down toward the violet in emission and up in the continuous background. The lines are often double and the components are not infrequently unequal, with fluctuating alternations of intensity. Struve's hypothesis of a rotating envelope, varying in extent from time to time, should explain much of this, but a good deal remains puzzling. Study of spectroscopic and especially eclipsing binaries which show emission lines should be instructive; but the best known of these is the notorious enigma Beta Lyrae."

The Problem.

The whole problem of Be-stars may be crystallised into one question; in what aspects is a Be-star different from a B-star of the same spectral sub-division, and what are the physical causes and consequences of these differences?

The Emission Lines.

To begin with, the first difference is that Be-stars show emission lines, while B-stars don't. In the pre-ionisation and quantum theory days, it was stated, rather vaguely, that emission is produced in "chromospheric layers" surrounding the star. An important step in defining the emission process was taken by Rosseland (11) who advanced the theory of cyclic transitions in a field of diluted radiation (by which is meant magnitude as the mean life time of the excited state from which emission takes place.

It is clear that these two processes of emission, fluorescence and recombination, are favoured by a low pressure, diluted radiation and a high temperature. The last condition explains in principle why emission lines are observed more in hot stars; the first two imply that within a spectral division, those stars which are more luminous are more expected to have emission.

A detailed application of this theory was made by H. H. Plaskett (13) in discussing the spectrum of the stellar component of Z And. The intensity of the emission lines in this star could not be explained by the mechanism; but Plaskett suggested that for normal Be-stars the extension of the atmosphere is necessary and sufficient for the appearance of bright lines. The same question was discussed by Miss Payne (14), who considered in detail the ionisation and excitation potentials of lines present in different spectral types. The highest excitation is taken as a criterion for the star. Thus if an atom has its ionisation potential lower than the maximum excitation potential observed in the star, then it will show emission lines formed through recombination; unless the ionisation potential of the atom in question is very much below the maximum potential of the star, in which case the atom will be completely ionised. On the other hand if the ionisation potential of the atom is higher than the maximum potential the bright lines will be formed through fluorescence. Only those lines whose upper levels are directly connected with the ground level, or can be reached from a lower state by absorption, ~~only such lines~~ will appear in any strength.

Summarising, the physical mechanisms of fluorescence and recombination explain the presence of emission lines in Be-stars; there seems to be no escape from the conclusion that Be-stars have extended atmospheres. There are still three major questions about emission-line stars;

1. How this extended atmosphere is formed and supported?

2. Why some emission lines are narrow while others are broad?

3. Two types of variation of emission lines are observed, in one the intensity of the line varies relative to the continuous (E / C variation), and in the other the relative intensity of the two components of a double emission line varies (V/R variation). How can these variations be explained?

The Absorption Lines.

The second difference between emission and normal B-stars may be looked for in the absorption lines. Considering absorption lines in a Be-star, we recognise two sets, one originating in the normal reversing layer and the other due to the shell. These two types of absorption lines are formed under different physical conditions, and in two different atmospheres, which may not have the same rotation or translation velocity.

The excitation of absorption lines in the outer shells of stars was studied by Struve and Wurm (15). They consider that only those lines originating from a metastable level will appear in any strength in the shell, other levels will be depopulated as a result of the dilution of radiation. For HeI the levels 2^1S and 2^3S are metastable, while 2^1P and 2^3P

are not. This explains why the line 3965 ($2^1S - 4^1P$) will be relatively sharper compared with the diffuse lines. This was found to be the case in some stars, such as ϕ Per. and ζ Taur. Struve and Wurm calculated the distribution of atoms between six levels of HeI, 1^1S, 2^1S, 2^1P, 2^3S, 2^3P and the state of ionisation, using different dilution factors and temperatures. In principle, a comparison between the observed and the calculated relative intensities of these lines should furnish a rough estimate of the dilution factor. Rough estimates (15) give a dilution factor W = .01. Using the approximate expression $W = \frac{1}{4}\frac{R^2}{r^2}$, we get r = 5R, where R is the radius of the star and r is the effective radius of the shell.

The most prominent absorption lines of the shell are those of hydrogen. They appear as single strong sharp lines in the centre of the wide double emission. Every Balmer line consists of three components, ~~one~~ coming from $2S_{1/2}$, $2P_{1/2}$ and $2P_{3/2}$ levels. Only the first of these levels is metastable, therefore in the shell we should expect a displacement* of the absorption line formed by the three components. For $H\alpha$ the change in wavelength is only about .16A and still even smaller for other lines. This seems to be beyond detection and a comparison between the Balmer and Paschen lines in stars having outer shells compared with normal super-giants should be more obvious.

Similar arguments were used by Struve (16) to discuss the excitation of other elements in Be-stars and also in the spectrum of Nova Herculis. The discussion is of a qualitative nature since the sharp and wide components are super-imposed on each other.

* due to the strengthening of the 2 $S_{\frac{1}{2}}$ level

Difficulties are not lacking here:-

(1) Absorption lines of the shell are sharp; indicating that the shell does not rotate as a solid body with the photosphere. The emission lines are wide in the same star and their width was attributed to the rotation of the shell.

(2) Many stars have narrow absorption lines, presumably due to a shell, but they show no emission. Conversely some stars have emission lines without sharp absorption ones.

(3) Comparing two stars, one with and the other without, a shell, we should expect all lines coming from non-metastable levels to have comparable intensities in the two stars, while lines originating from metastable levels will be relatively stronger and sharper in the shell-star. This is contrary to observations. Non-metastable level lines are seen very much weakened in Be-stars. This led Struve and Wurm (p.108) to conclude,- "It is difficult to avoid the conclusion that the shell produces a screen of continuous emission which weakens the underlying absorption lines."

This leads us to the general absorption and emission of the assumed shell.

The Opacity of the Extended Atmosphere.

So far we have assumed that the extended atmosphere is transparent to its own radiation. This cannot be a good approximation even for planetary nebulae (17). The importance of the continuous absorption in the shell should be expected and recognised. This has an important bearing on the reddening of early-type stars. Williams has found (18) that the residual colour excess (after allowing for the distance effect) of Be-stars seemed to be related to the intensity of the hydrogen emission, the stars with stronger emission lines being on the average lower in colour-temperature. Colour-temperature results for γ Cas. measured by Greaves and Martin (19) are in accordance with this conclusion. The brightening of the star in 1936 was accompanied by a strengthening of the emission lines and by a fall in the colour-temperature.

From the intensity of the wide hydrogen absorption lines, Struve (20) deduced that the outer shells of some stars have a considerable optical thickness. Consider the wings of absorption lines in the three stars:-

(1) H.D.190073, typical broad lines with wings showing Stark effect;

(2) 17 Leporis, wings are too weak for spectral type, but the classification is certain;

(3) P Cyg., no trace of wings.

The differences are attributed to different opacities of the shells which are assumed round these stars. In P Cyg. the shell is opaque, transparent in H.D.190073, and intermediate in 17 Leporis. A rough calculation by Struve (20) shows that the general absorption by hydrogen, metals and free electrons may be sufficient to account for the continuous absorption observed. Generally speaking, an increase in the coefficient of continuous absorption will cause an in-flow of radiation and consequently a weakening of line-emission. On the other hand it may be expected that emission will be stronger for more extended atmospheres.

It was suggested by Woolley (21) that reddening of B-stars may be due to continuous emission extending over the visual region, and coming to a maximum at the head of the Paschen series. The problem of reddened B-stars is by no means finally settled, but as far as Be-stars are concerned it may be mentioned that their tendency towards concentrating in limited areas (1) strengthens the interstellar origin of absorption; while the existence of extended atmospheres round these stars strongly suggests a stellar origin. Stellar models with extended photospheres studied by Kosirov (22) and Chandrasekhar (23) may be able to explain some of the observed anomalies.

Origin and Motion of the Atmosphere.

The problem of Be-stars cannot be considered as solved until the origin of the nebular envelopes is explained, and at the same time, the contours of emission lines and their variations interpreted. The origin, stability and motion of this stellar model represent a dynamical problem of the first magnitude. Qualitatively, it is seen that radiation pressure and rotation are the two important factors concerned. Attempts to obtain more precise information than from qualitative reasoning, show that the problem is so complicated as to be mathematically intractable except under the most drastic simplifications. Even then, many outstanding facts and observations remain unexplained.

From statistical studies of rotational broadening of absorption lines in different spectral types (24) it is found that A-type stars have rotational velocities larger than B or F stars. Rotation is directly observable in eclipsing binaries (25), and to some extent by Carroll's analytical method (26). Rotation is indicated by distorted "dish-shaped" contours of absorption lines (H and He lines are excluded because of their susceptibility to Stark effect). The fact that spectroscopic binaries so often have broadened lines, and almost invariably so, when the orbital period is short, is the strongest evidence of explaining the distortion by rotation and not by some other cause.

The instability due to axial rotation is the starting point of Struve's hypothesis of the origin of the emitting layers (27). It is based upon the correlation of emission and absorption line-widths, and upon the assumption that "dish-shaped" contours are caused by rapid rotation. The mean purpose of Struve's hypothesis, apart from providing a mechanism by which matter comes out of the star, is to explain the width of emission lines as Doppler broadening. Many objections were raised against this theory (28 & 29); the most serious of which, perhaps, is the following. The constancy of angular momentum demands that VR = constant, R is the distance from the axis and V is the linear rotation and velocity. This leads to very small velocities for the outermost layers which are about a hundred times smaller than those needed to explain the width of the emission lines. There seems to be little doubt that the outer shells rotate slower than the star; (30) and therefore the proportionaility of emission line-widths with wavelength (31) cannot be attributed to rotation.

The effect of radiation pressure was discussed by Gerasimovič (29) who revised previous calculations by Johnson (32). Gerasimovič found that for temperatures higher than 10,000, the acceleration of hydrogen atoms (j rad) acquired by absorbing radiation beyond the Lyman limit is given by

$$j_{rad} = \frac{p_e}{(KT)^{3/2}} \times 2.6\times10^{-17} \quad C.G.S. \qquad p_e = \text{electron pressure.}$$

There is also an acceleration due to absorption of $Ly\alpha$

Under suitable conditions, an outward movement of the atoms begins; and after some time there exists a flux of radiation towards the centre, which may, or may not, be sufficient to stop or reverse the motion of the atoms. Gerasimovič considers two cases:-

(1) A permanent non-static atmosphere is developed in which a continuous stream of outflowing matter takes place. Gravitation is too small to check it.

(2) g may be too small to form a static atmosphere, but large enough to prevent a permanent non-static one. In this case the inward flux will stop expansion which may recommence. This may give rise to cyclic variations in the position and intensity of the emission lines.

Gerasimovič maintains that Bse stars correspond to the first case, provided that the optical thickness of the atmosphere is small enough to be compatible with the weak emission lines observed in these stars. Bne stars correspond to the second case. He considers this to be in agreement with Curtiss and McLaughlin's observation (33) that Bne stars are more frequently variable in emission compared with Bse stars; and with Merrill's observation (34) that in general Balmer emission is much weaker in Bse ~~stars~~ than in Bne stars. Rotation, from Gerasimovič's point of view helps the birth of a non-static chromosphere and may be needed to maintain the atmospheres of Bne stars.

It is noticed that the theories advanced by Struve and Gerasimovič do not contribute much in explaining the variation of emission lines neither in absolute (E/C) nor relative (V/R) intensities. An attempt in this direction was made by McLaughlin (35) who combines rotation, pulsation and temperature fluctuations, to represent the V/R variations. Some of the fundamental arguments in McLaughlin's model are very doubtful (36). Gerasimovic (36) tried to explain the variations along his model and obtained interesting results. Swings and his co-workers (37) emphasised the importance of the distribution of the physical conditions in the atmosphere, especially the rate of change of pressure with optical depth. Almost in all these cases, the writer goes so far in assuming properties for his model to render observational tests difficult or impossible, or he may, in trying to explain one point raise many difficulties.

On the observational side, a recent paper byCherrington(38), if confirmed, should be of great importance. Cherrington found that 72% of the B stars examined by him exhibited evidence of a "super-shell." This evidence is in the form of sharp absorption lines together with wide ones in the same spectrum. He found also a definite velocity difference between these two sets of lines; and in every case the difference was in the direction of greater velocity of approach of the sharp lines, indicating an expanding shell. Cherrington did not publish his full measurements; and it is well known that rdaial velocity measures of wide-absorption lines in early-type stars are liable to large errors, and also the identification of sharp lines may not be very certain. An unsuccessful attempt to identify these lines, if they were really present, and measure their radial velocities, was made by the writer. Incidentally, it may be noticed that all the stars measured by Cherrington are Bn and Bne stars, which are, according to Gérasimovič, not steadily expanding.

Statistical Remarks.

The number of Be stars is round about 400 (1). They are distributed among all sub-types, with a marked tendency towards B0 - B3. The ratio of the number of emission stars to the total number of normal stars of the same class is certainly less in Bo and B1 sub-divisions than in B3. It was shown (39) that this is a real effect and cannot be due to errors in classification. As far as can be ascertained there is no definite explanation for this distribution.

There is some evidence that Be stars are, on the average, more luminous than main-sequence stars without emission. We are now less sure about this conclusion than ten years ago. In 1927 Gerasimovic (40) found that B3e stars are 2.4 magnitudes brighter than B3 stars. McLeod (41), using radial velocity and proper motion data, reduced the difference to 0.4 mag. Again Gerasimovic (42), using five different methods, concluded that Be stars are 1.5 mag. brighter than B stars. Another result by Mineur (43) gives a difference of 0.3 mag.

The discovery of Be-stars is not complete for fainter magnitudes; and therefore the distribution of Be-stars in magnitude-intervals is not fully significant. Galactic distribution of Be stars is for a first approximation the same as normal B stars, Emission-line stars have a somewhat stronger tendency to appear in condensed groups. Normal stars exhibit a systematic departure from the galactic circle, attributed to a "local system" or flattened condensation of stars in the neighbourhood of the sun. Be stars brighter than 6.25 mag. show the effects of the local system, although less strongly than do the corresponding non-emission stars; but for Be-stars in the interval 6.25 - 8.25 mag., local system characteristics are not clear.

Statistical studies of Be-stars are limited to a certain extent by their relatively small number.

RESUME.

From the theoretical point of view, emission lines in early type stars are formed through Zanstra's method of recombinations and Rosseland's cyclic transitions. Emission takes place in a nebular shell round the star, which may be rotating, pulsating or steadily expanding. The shell may give rise also to absorption lines, and to absorption and emission in the continuous. Theoretical studies are on the same lines as in the general nebular problem. Additional complications are caused by the proximity of the star, the relatively small extent of the shell, as well as the mechanical effects of its motion. Recent investigations by Kurihara (44) seem to be promising.

As regards observation, there is still a great deal to be made. Very little is know about the profiles of emission lines. According to Gerasimovic, there should be a difference between the profiles of emission lines in Bn and Bs stars. It had been said (27) that the frequent variation of emission lines constitutes the most vulnerable point in the rotational hypothesis. It is only natural to attack from the vulnerable point.

There are many questions in connection with the absorption lines formed in the "inevitable" surrounding shell. These lines have been, so far, studied qualitatively. Their intensities, relative to the intensities of the reversing-layer-lines and relative to each other, should yield many important facts about the physical conditions in the shell. These lines are more frequent in the ultra-violet region; the recent papers by Swings & Struve (45) are very full of important data. The infra-red region shows the Paschen lines which may be compared with the Balmer lines, so as to reveal any effectddue to the metastability of the level.

Observations of the continuous spectra of Be-stars are very much needed. According to Woolley emssion may be extending from the Paschen limit to the visual region. Comparison between normal- and emission B-stars absorption lines to detect any incipient emission by fluorescence (46) provides a fruitful field of investigation. The same applies to the variation of colour-temperature with emission line intensities. In the ultra-violet continuous, there is the Balmer discontinuity, which was suggested (47) to be smaller in emission-line stars. It is also suspected that the shell may have its own Balmer discontinuity, besides that of the reversing layer. Barbier and Chalonge (48) observed a double Balmer limit in the Be-star Tauri; this was not confirmed by Swings and Struve (46).

The radial velocity measurements of the shell-lines made by Cherrington (38) must be considered to be of a preliminary character, and more measures are needed to establish his results. Attention should be paid to the binary and eclipsing B-stars, since there is good reason to believe (45) that the binary nature of a star stimulates the process of shell-formation.

The Material, Observation & Results.

General Remarks.

All the material used in the following study was taken with the 36" reflector of the Royal Observatory, Edinburgh, by the members of the staff, and subsequently handed to the writer. The ϕ Per.-plates (in the Second Part) were measured first. It was hoped that a complete set of standardised double-prism plates of this star could be secured, so as to cover the whole period. Owing to unfavourable weather and the present emergency, this programme did not materialise. Measurements of the available plates are presented in the Third Part. The total intensities of SiIII-lines in early B-type stars were measured last.

There have been two unsuccessful attempts. The first was to measure the radial velocities of sharp and wide lines in Bne stars and observe any systematic difference. The second was to classify the lines of different elements within the same spectrum according to sharpness. This was a natural step to define what is meant by a narrow line. Three stars were measured, but owing to the limited resolving power, the work was not finished. Some of the by-products are given in the Third Part.

Corresponding Lists of References will be found at the end of each Part.

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(46) For a detailed study of fluorescence see A. Pannekoek M.N. 95, 732, 1935.

(47) D.BARBIER & D.CHALONGE: C.R. 207, 895, 1938.

(48) D.BARBIER & D.CHALONGE: Ap.J. 90, 628, 1939.

PART II.

The Be- Variable Spectrum of ϕ Persei.

Abstract.

Radial velocities of central hydrogen absorption lines were measured on 23 quartz-prism spectrograms, covering the star's period of variation. The new observations agree well with the period given by Dustheimer, but it is probable that the decrease in the velocity range is real.

The secondary variation is discussed. It is likely that the extent of this variation increases for lines of shorter wave-length. The observations are compared with Schiefer's results.

The variation of the number of Balmer lines with sharp central cores is examined. It is shown that this variation is not likely to occur in the same atmosphere, and therefore the binary hypothesis seems necessary.

THE Be VARIABLE SPECTRUM OF ϕ PERSEI.

R.A. (1900) $1^{h}37^{m.}$ Spectral type Bone. Mag. 4.2

Decl.(1900) $+50^{\circ}11'$. H.D. number 10516.

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This spectrum shows emission lines of hydrogen and ionised iron. Each hydrogen line consists of a very broad absorption band upon which is superimposed a narrower emission. The emission is divided into two components (V and R components) by a sharp central absorption. Emission is strongest for $H\alpha$ and decreases with wave-length. FeII lines are broad and double. Other elements show absorption lines which are usually broad and shallow. The radial velocity is cyclically variable in a period of 126.626 days. Almost all of the above mentioned features are variable.

Historical Introduction.

Early observations of the spectrum were mainly concerned with the appearance of bright lines, notes on which were given by Pickering (1), Espin (2), Campbell (3), Vogel and Wilsing (4), and Maury (5). The radial velocity variation was first observed by Campbell (6), who announced the star as a spectroscopic binary.

Determination of the orbital elements was the subject of three studies by Ludendorf (7), Cannon (8), and Jordan (9), each involving well over a hundred spectrograms. The velocity curve could not be represented by simple orbital motion. Besides the usual maximum and minimum velocity, there exist, on the descending branch of the velocity curve after maximum positive velocity, a secondary minimum followed by a secondary maximum (See fig. I, a)(p.34)These secondary maximum and minimum are referred to as the secondary variation. Cannon (8) suggested a model for the system consisting of a bright star, revolving about a central dark one, thus giving rise to the principal variation. The bright star is accompanied by a satellite, whose revolution about the bright star in half the period is considered the cause of the secondary variation. This model did not fit the observations.

Variability of emission was first mentioned by Merrill (10), and later discussed in detail by Curtiss (11), (12), especially with regard to the relation between width of emission lines and wavelength. Variation of emission complicated the models based on orbital motion. Pulsation was then introduced as a possible mechanism causing cyclic variations.

Lockyer's investigations (13), (14), threw considerable light on the problem. He studied the relative intensities of emission and absorption lines, using eye-estimates and a graduated wedge. His most important result was that the variation of the ratio of the intensities of the emission components is periodical, and corresponds to the position of the central absorption, in such a way so as to make the maximum positive velocity correspond to maximum intensity of the violet component relative to the red one, and vise versa. Thus Lockyer's curve of V/R is exactly similar to the radial velocity curve. These characteristics of bright lines were observed in a group of stars, which were called "ϕ Persei Variables", by Curtiss (15).

The last radial velocity studies of the star were carried out by Dustheimer (16) and Schiefer (17), both at Michigan. Dustheimer derived an accurate period based on 588 observations. He also discussed the variation of the helium and calcium lines, both in intensity and velocity. Schiefer's plates were calibrated with a tube sensitometer, and thus his intensity measures are entitled to a larger weight. He confirmed previous observations, and advanced a new model for the system, consisting of two components in orbital motion, each rapidly rotating and having an extended atmosphere. To allow for the secondary variation, he assumed the fainter star to be pulsating. This model explains more than its predecessors do, but it is correspondingly more complicated, and furthermore, it does not explain all the observations. From his measurements Schiefer concluded that the secondary variation for Balmer lines decreases progressively and lasts shorter time for longer wavelengths. This is attributed to a disturbance, which is supposed to affect the lower atmospheric levels of the star more than the higher ones, assuming that the lines of shorter wavelengths originate at lower levels. This result has never been observed again, and it was thought profitable to check it.

In recent years, interest in the star was renewed, mainly in connection with the general theories of bright-line formation and variation. Most of these studies are, therefore, carried out photometrically.

The main purposes of the present study are :-

1. to test the constancy and accuracy of the period and the velocity curves;
2. to re-observe the secondary variation and check Schiefer's conclusions about it;
3. to study the variation in intensity of the central hydrogen absorption along the star's cycle.

There is a gap, 30 cycles wide, between the present data and the last observations published. Thus a new velocity curve of the star should reveal any variable feature.

Observation and Measures.

The material consists of 23 spectrograms taken with the 36" reflector of the Royal Observatory, Edinburgh, in connection with a universal type spectrograph. One plate was taken with the 2-glass prisms combination, 2 plates, with single prism, and the rest with the quartz prism. The linear dispersion on the plate at $H\gamma$ for different combinations is as follows :-

single prism	96 A / mm.
two prisms	29 A / mm.
quartz prism	54.5 A / mm.

The plates are usually in good focus throughout a region extending from the end of the Balmer series on the violet side up to $H\beta$ on the red side. The 2-prisms plate is focussed for the region H_γ - H_α , $H\delta$ is almost out of focus. The comparison spectrum was supplied by an iron arc, and was usually introduced~~, usually~~, intermittently during exposure. The plates were not standardised, and thus the meterial is not suitable for any accurate photometric measurements.

The spectra were measured in a travelling-carriage Hilger micrometer $\frac{\text{L } 18.301.}{29.201}$ Four independent settings were made on each stellar and comparison line, 2 in the direct way and 2 after reversing. Hartman constants were derived for each plate separately. Two sets of constants were generally computed, one to cover the $H\beta$ - $H\gamma$ region and the other for the region of shorter wavelength. A correction curve was then derived from other comparison lines to correct the calculated wavelengths.

Wavelengths given by Burns (18) were used. They provide a set of homogeneous and complete values of the arc lines in the region studied. The differences between Burns' values on the one hand and the more accurate ones used in the Victoria system of radial velocity determination, and those adopted by the I.A.U. (19), on the other hand, were discussed by Harper (20). These differences do not amount to anything more than a fraction of a km./sec., which is beyond the accuracy of measurement.

A search was made for curvature effect. This was done by measuring 2 plates, having arc comparison spectra in the centre and on both sides. No systematic difference could be seen between the settings on the centres and the ends of the lines. Curvature of field must be present to some extent, but it is too small to affect the present measures.

The spectrograph is kept at constant temperature by a thermostat and is continuously ventilated by a fan. This renders a change of temperature improbable; and the fact that intermittent exposures of the comparison were made, also reduces the error of temperature.

The most serious source of errors is the quality of the spectra as regards exposure and line width. Seven plates had to be rejected because of over-exposure either in the star, or the comparison, or both.

To have an idea about the radial-velocity determined here compared with other observatories, a set of spectra of a star of constant radial velocity was needed. In the material available only one star was found, namely i Her., which had only 2 spectra taken on the same night. The measured velocity was found to be

-16.3 Km./Sec.

compared with the Victoria-value of -17.0 Km./Sec.

and that of the I.A.U. (21) -18.0 Km./Sec.

This agreement is actually less assuring than it seems to be, because it is based on 2 measures, and because the star was

TABLE I - Radial velocity of absorption lines of hydrogen in Km./Sec.

No.	Plate	Date	Phase (days)	H_γ	H_δ	H_ϵ	H_ζ	H_9	H_{10}	H_{11}	Mean H	$H-H_\gamma$	$H-H_\delta$	$H-H_\epsilon$	$H-H_\zeta$	$H-H_9$	$H-H_{10}$	$H-H_{11}$
1	70/36	Dec.29.8	6.7	-7	-2	-7	-20.0	-22	-20	-20	-16.4	-9	-14	-9	3.6	+3	+4	+4
2	74/36	Dec.31.8	8.7	-8	-1	0	- 9.9	+ 2	+ 5	-10	+ 3.4	11	4	3	13.3	-1	-2	13
3	110/35	Dec,20.9	11.7	-8	-5	+1	- 6.3	00	+10	-11	- 1.6	7	4	-3	4.7	-2	-11	+10
4	(116/35)	Dec.28.9	14.7	34	31	30	+27.0	39	24	23	+35.2	1	4	5	8.2	-4	11	12
5	2/37	Jan. 6.8	14.7	17	22	23	+12.3	32	15	20	+19.8	3	-2	-3	7.5	-12	5	0
6	7/37	Jan. 7.8	15.8	15	12	5	+ 0.1	16	9	22	+10.4	-5	-2	5	10.3	- 6	1	-12
7	(14/37)	Jan.16.9	24.8	44	39	40	+16.4	31	41	27	+35.0	-9	-4	-5	18.6	+ 4	-6	8
8	(8/38)	Feb. 9.8	33.9	15	8	22	- 8.0	--	--	--	+14.8	0	7	-7	22.8	- -	--	--
9	7/36	Jan. 6.8	40.6	14	9	12	-13.3	-17	-30	15	+ 0.5	-14	-9	-12	13.8	17	30	-14
10	13/36	Jan.22.8	44.7	17	32	-10	-22.0	32	7	-13	+ 8.7	- 8	-23	+19	30.7	-23	2	22
11	16/36	Jan.23.8	45.6	-3	-10	- 6	-28.2	-23	10	-40	-12.2	- 9	- 2	- 6	16.0	-11	-22	28
12	23/36	Feb. 3.8	56.6	33	+38	20	+ 5.8	7	33	20	+15.9	-17	-22	- 4	10.1	-19	-17	-4
13	x90/37	Oct.29.0	56.7	-3	23	-9	-13.5	9	4	00	+ 2.6	6	-20	12	16.1	- 6	- 1	-2
14	30/36	Feb.11.8	64.4	-16	3	16	+ 2.5	16	00	16	+10.3	26	7	-6	7.8	- 6	10	-6

x Measured velocity of H_{12} is -6 Km./Sec., not included in Table I - A.

TABLE I - (Contd.)

No.	Plate	Date	Phase (days)	H_γ	H_δ	H_ϵ	H_ζ	H_9	H_{10}	H_{11}	Mean H	$H-H_\gamma$	$H-H_\delta$	$H-H_\epsilon$	$H-H_\zeta$	$H-H_9$	$H-H_{10}$	$H-H_{11}$
15	x55/39	Dec. 6.9	65.8	9	6	--	--	--	--	--	+13.5	5	8	--	--	--	--	- -
16	93/37	Nov.12.9	72.1	-1	-7	12	-18.3	-2	-10	-13	- 2.4	-1	-5	-14	15.9	0	8	11
17	73/35	Oct.17.0	73.4	24	-30	-19	-16.7	9	--	--	- 4.2	-29	26	23	12.5	-13	--	--
18	96/37	Nov.20 5	79.6	-1	1	6	- 9.3	16	19	-12	+ 4.7	6	4	- 1	14.0	-11	-14	+17
19	(78/35)	Oct.31.9	87.4	--	-21	-38	-54.4	-26	-33	-53	-33.0	--	-12	5	21.4	- 7	0	20
20	102/37	Dec.15.8	104.6	-11	-16	--	-34.8	-36	8	--	-13.2	-25	-3	--	27.0	23	-21	- -
21	106/37	Dec.31.8	120.5	-19	- 3	-19	-45.7	-14	-27	-10	-18.7	0	-16	0	21.6	- 5	8	9
22	1/38	Jan. 5.8	125.5	- 4	- 4	- 2	-22.0	- 8	2	-15	3.4	7	7	5	25.4	15	1	18
23	63/36	Dec.22.6	126.4	- 6	17	10	-20.2	- 9	-8	0	-6.7	-1	-24	-17	13.5	2	1	-7
		Mean										-2.5	-4.1	00	+15.2	-2.0	00	+4.0

x a 2-prism plate in which $H\beta$ was measured

( ) indicates plate measured twice [redacted].

later reported as variable by Harper and by Edwards (22).

But the present study is mainly concerned with relative velocity changes, and absolute values are not essential. The reduction to the sun was computed using Herrick's tables (23).

All the spectrograms were secured by Mr J. Storey. The dates are given ~~be~~low in Table I. (pages 30 and 31).

Results and Discussion.

Lines measured. The only reliable measures were those of central absorption of hydrogen lines. An attempt was made to measure some HeI and FeII lines, but they were seen only on a few plates, HeI lines generally as hazy undefined absorptions, and FeII lines as wide faint double emission. Lockyer (13) and Edwards (24) reported sharp double lines of FeII on the Sidmouth objective prism spectrograms, a totally different description from that of Jordan (9), and from what is seen on the present plates. Such a discrepancy may not be real, it may be simply due to photographic effects, which have been explained by Frédette (32).

The radial velocity measures are given in Table I. The second column gives the plate-number and year, the third gives the date and U.T. The phase, given in days, in column 4, is calculated using a period of 126-626 days, referred to a zero phase at November 18.00, 1925, U~~G.~~M.T. These values were derived by Dustheimer (16).

TABLE I - A

Measured radial velocities in km./Sec. of Balmer lines after H_{11}

(Serial number in the first column refers to the first column in Table I).

No.	H_{12}	H_{13}	H_{14}	H_{15}	H_{16}	H_{17}	H_{18}	H_{19}	H_{20}	H_{21}	H_{22}	H_{23}	H_{24}
1	-25	-19	-18	-27	-18	-22	- 8	-14	- -	- -	- -	- -	- -
2	+21	+ 7	-15	+ 9	+ 5	- 5	- 3	-14	+10	+21	+01	+27	- 8
3	+ 5	+ 8	+ 5	-19	-13	- 4	- 7	-10	-25	+16	+32	+ 9	-12
4	+50	+26	+51	+32	+34	+32	+48	- -	- -	- -	- -	- -	- -
5	- 4	+26	+33	+36	+20	+17	+22	- 3	- 5	+50	- -	- -	- -
6	+16	+20	+18	+ 9	+ 5	+ 6	- 1	- 4	0	+17	+13	+ 8	- -
7	+41	+10	+24	+24	+64	- -	- -	- -	- -	- -	- -	- -	- -
10	+ 8	- 7	- -	- -	+17	- -	- -	- -	- -	- -	- -	- -	- -
12	+05	-26	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -
14	+ 5	+ 8	+26	+ 8	+ 1	+33	+20	+12	- 4	- -	- -	- -	- -
16	+ 4	- 8	- 3	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -
18	-21	-20	+13	+21	- -	- -	- -	- -	- -	- -	- -	- -	- -
19	-22	-16	-79	-50	- 7	- -	- -	- -	- -	- -	- -	- -	- -
21	-36	-38	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -
22	+ 9	+15	+15	+26	+ 3	+ 1	- -	- -	- -	- -	- -	- -	- -
23	-14	-14	-13	+ 5	-10	-16	- 7	+ 3	-14	-31	- -	- -	- -

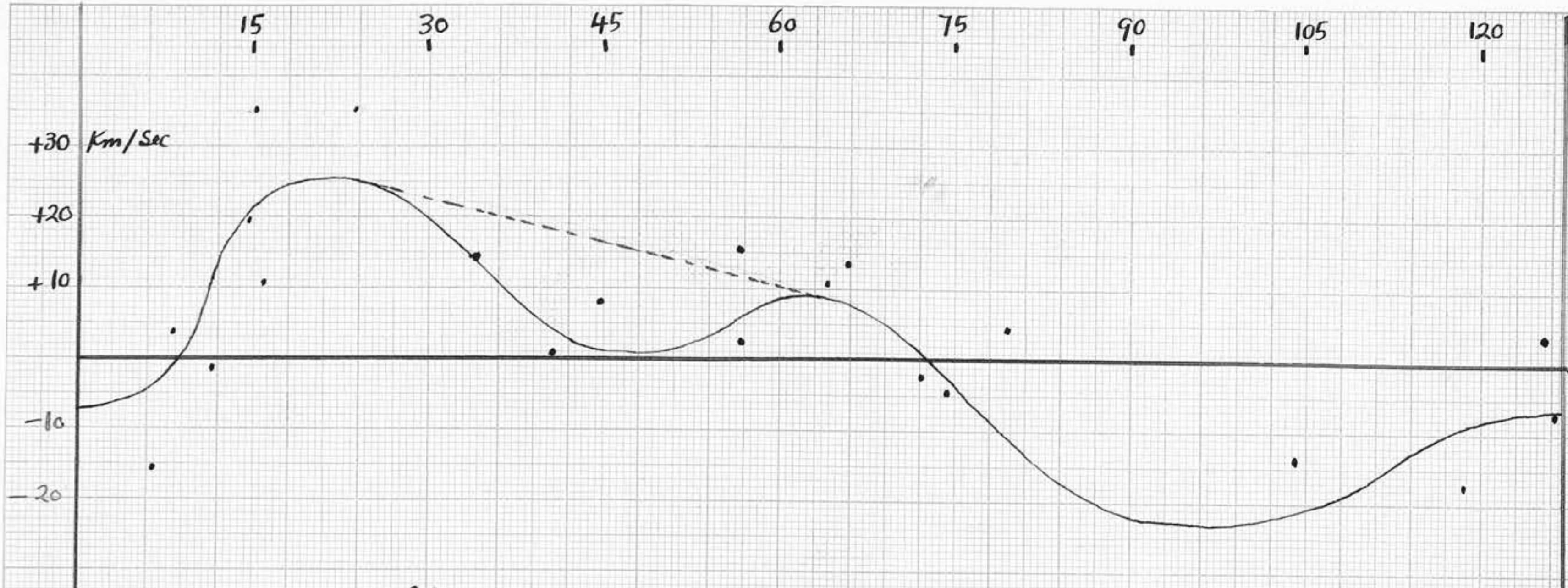


Fig 1-a - Mean Hydrogen Velocity Curve.

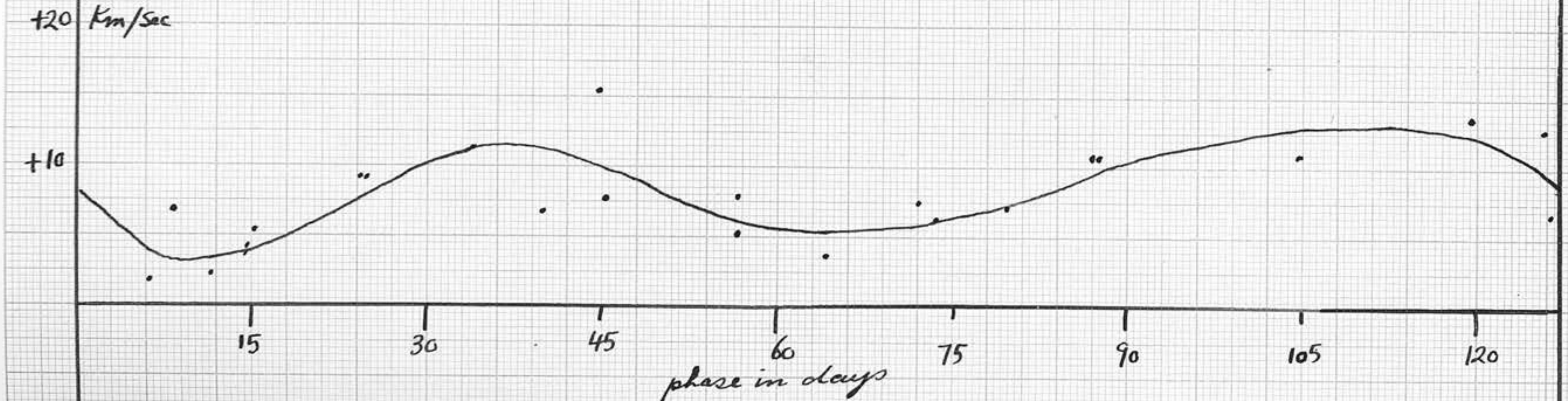


Fig 1-b - Change of (H-Hξ) with Phase.

The next seven columns give the radial velocity of hydrogen lines. H_ζ is seriously blended with the HeI - principal triplet line at 3888.65 A, and the tabulated velocity is that of the blend, written H_ζ for abbreviation. In the column headed "Mean H" are given the means of all hydrogen lines, excluding the blended H_ζ and including any lines measured after H_{11}. Radial velocities of lines beyond H_{11} are given separately in Table I - A. (page 33)

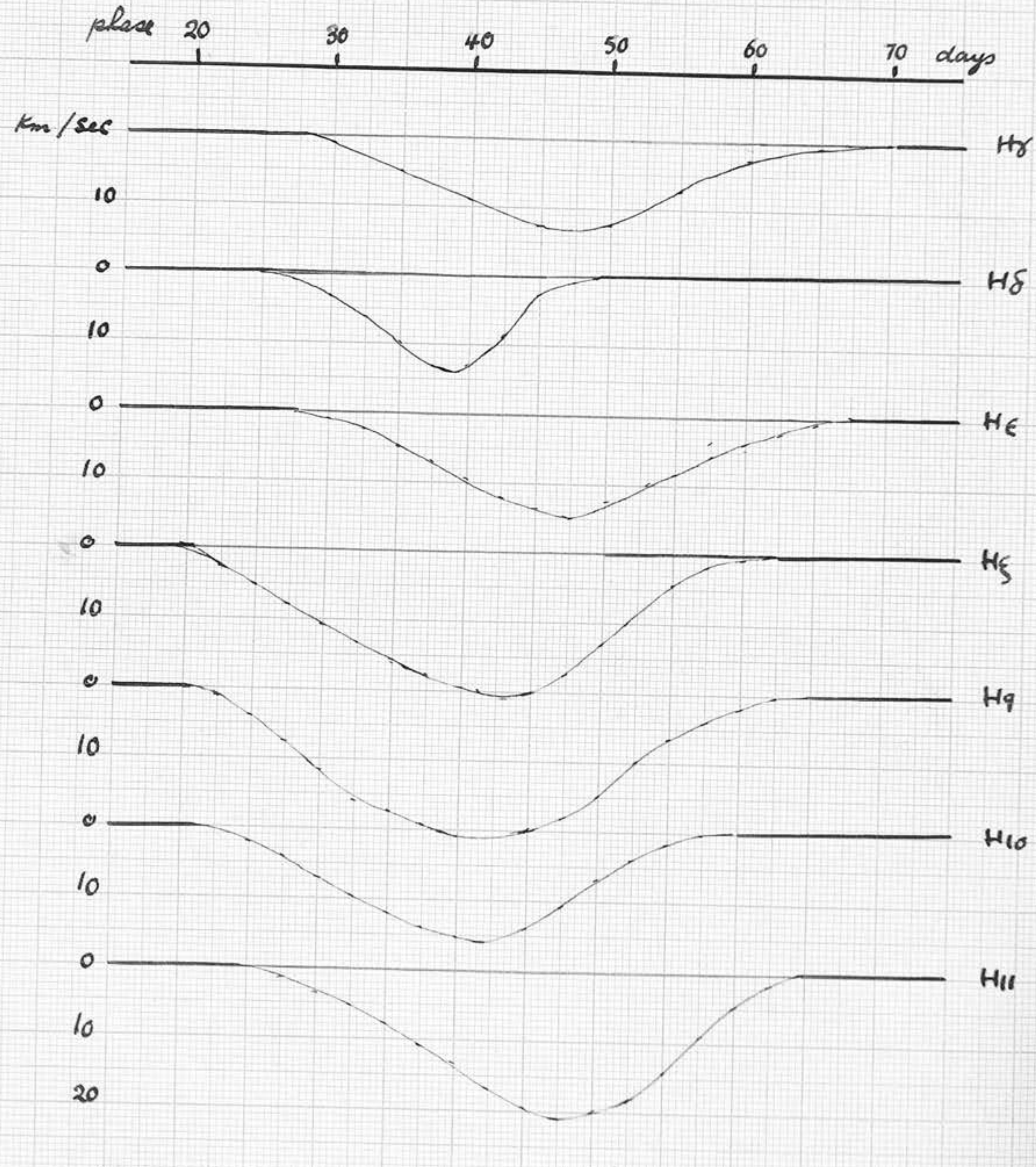
The differences between the mean hydrogen velocity and the velocities of separate lines are tabulated in the last seven columns. The mean of these differences is written at the bottom of the table. This mean, which is only significant for $H - H_\zeta$, will be discussed later.

Velocity-phase curves were drawn for each line separately and for the mean. Maximum positive velocity is reached at about 20 days, followed by a secondary minimum velocity at 45 days, then a secondary maximum at 62 days followed by a principal minimum about 100 days. All curves show the same features, but they differ with respect to the secondary variation.

The Secondary Variation.

Schiefer isolated the secondary variation by assuming it to be a negative velocity below a line connecting the principal and secondary maxima (dotted line in fig. Ia). A Similar procedure was followed here, and the isolated variations were obtained for every line. The result is shown graphically in Fig. 2, which corresponds to Fig. 4 (p.574) in Schiefer's paper.

Fig 2.



Isolated Secondary Variations.

From his curves Schiefer concluded that, for shorter wavelengths, the range of maximum secondary variation is larger and the phase of this maximum is earlier. Actually, Schiefer mentioned that the variation begins earlier and lasts longer for shorter wavelengths; but the beginning & end points of the curve are not sharply defined, and it was found more accurate to use the phase of maximum secondary variation in comparing the present data with Schiefer's rather than the beginning and end points. Values of the range and phase of maximum secondary variation for every line are read from Fig.2 and plotted in Fig.3 against the quantum number of the line. From Fig.3,a, it is seen that the maximum secondary variation increases slowly towards shorter wavelengths. The only notable exception is H_{10}, but the scatter of points is reasonably small. The rate of increase is much smaller than Schiefer's. It is also noticed that the extent of variation is smaller for the Edinburgh measures.

Fig. 3,b, shows a similar comparison for the phase of maximum secondary variation. The scatter is larger than in Fig. 3,a. The Edinburgh mean curve is a straight line parallel to the wavelength axis, indicating that the phase of maximum variation is the same for all lines, namely 45 days. Even if we reject the two points corresponding to H_δ and H_{11}, which are most discordant, still the resultant line will have a smaller slope compared with Schiefer's.

The conclusion is that the cause of secondary variation, whatever it may be, affects lines of short wavelength more than longer wavelength ones, and that the maximum effect occurs almost at the same phase. These observational facts have to be explained by any proposed model of the system.

Fig. 3-a & 3-b.

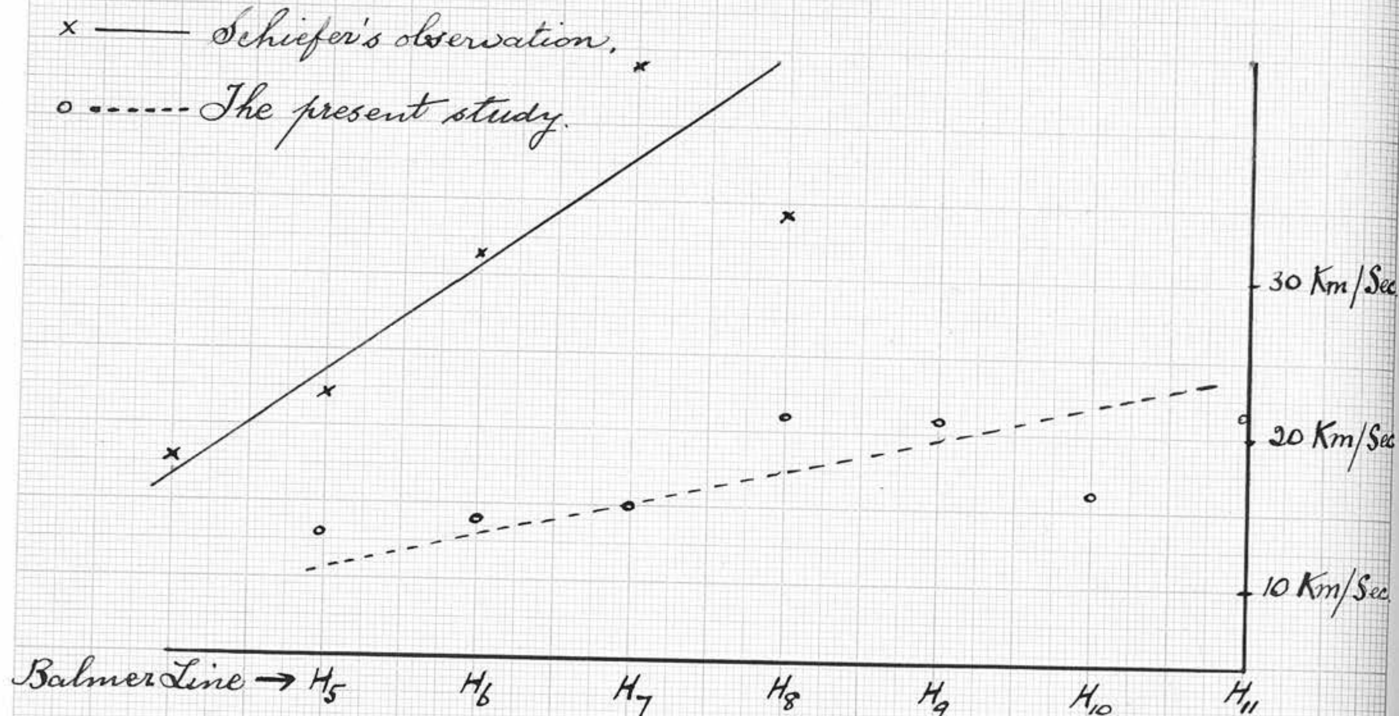


Fig. 3-a. Maximum Secondary Variation for Different Lines.

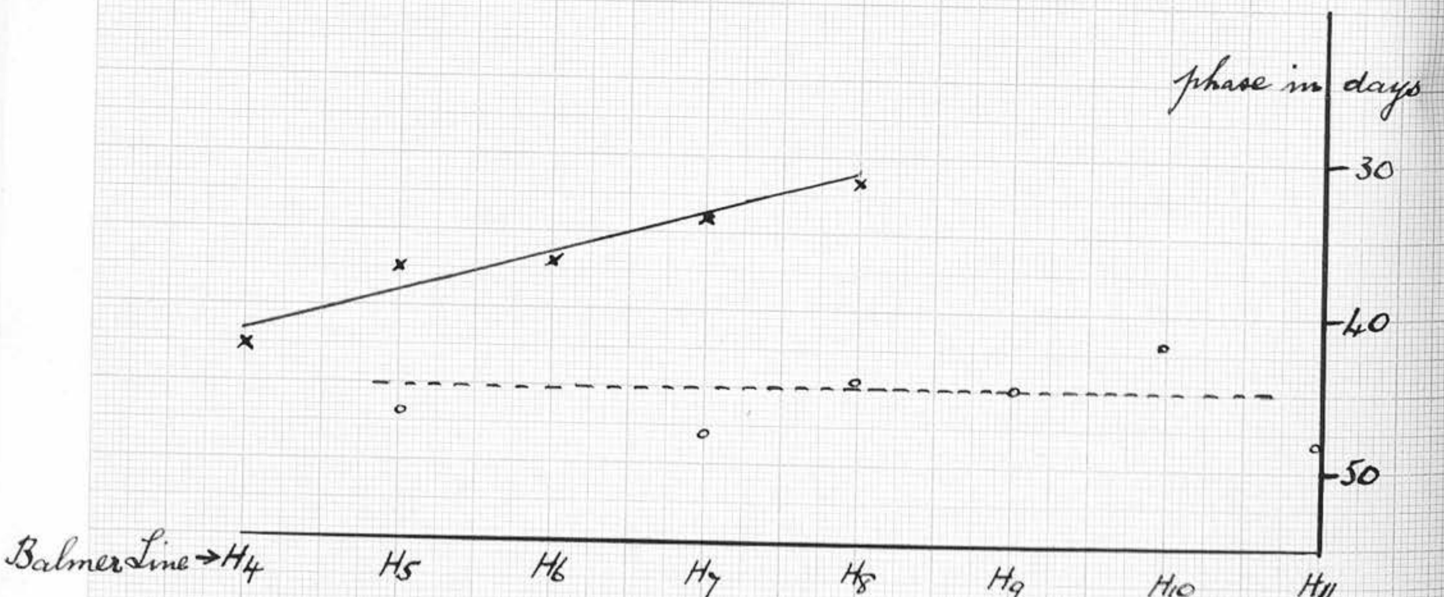


Fig. 3-b. Phase of maximum Secondary Variation for Different Lines.

The Period.

The present observations are in the average 30 cycles later than the last measures published, and thus they are suited to test the accuracy of the period. Such an inaccuracy, or a change, if existed, would be shown as a variation in the phases of maximum or minimum velocity compared with previous observations. The radial velocity of the star has been studied during two main periods, the first about 1909 by Ludendorf, Cannon and Jordan, and the second about 1926 by Dustheimer and Schiefer. Thus it seems reasonable to have previous observations in two corresponding groups. This is done in Table II, where the measures of Ludendorf, Cannon, and Jordan are put together in group I, and those of Dustheimer and Schiefer in Group II.

TABLE II - Phases of Maxima & Minima.

Observer	1	2	3	4	5	6	7	8
Phase of maximum velocity	26	25	20	23	25	24	24	22
Phase of minimum velocity	97	98	97	100	102	97	101	100
Phase of secondary maximum	59	57	56	63	62	57	63	60
Phase of secondary minimum	48	47	46	53	51	47	52	45

(Phase in days)

1 = Ludendorf's value
2 = Cannon's "
3 = Jordan's "
4 = Dustheimer's "
5 = Schiefer's value
6 = Group I
7 = Group II
8 = Edinburgh value

Ludendorff used $H\gamma$ only, Cannon measured $H\beta$, $H\gamma$, $H\delta$ and $H\epsilon$, Jordan $H\gamma$, $H\delta$ and $H\epsilon$, Schiefer $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$ and $H\zeta$, while in Dustheimer's and also in the present study all measureable lines were used.

Dustheimer derived the period of 126.626 from all observations available to him then; these observations are based on different number of Balmer lines. Any possible difference between curves derived from different lines is thus neglected, and Dustheimer has assumed, implicitly, that all lines have the same curve, as far as the principal maximum and minimum, are concerned. We have used Dustheimer's period, and thus accepted his assumption. This means that we can compare curves derived from different numbers of lines. This is done in Table II, above. The purpose of comparison in Table II is to test the period, and see how far it is able to represent the present observations. The agreement between phases of principal maximum and minimum in groups I, II and Edinburgh, indicates that the Dustheimer's period fits our observations closely. Had we found systematic differences between these phases, we would have attributed them either to inaccuracy or variability of the period on the one hand, or to the assumption explained above on the other hand.

The agreement between secondary maximum and minimumphases is not complete, but there is no systematic change detectable. The question of the variability of secondary minimum and maximum with phase cannot be settled now.

The mean deviations of velocity of different lines from the mean hydrogen velocity, are given at the bottom of Table I. They are not considered significant to justify any conclusion about the difference between curves of different lines.

The Range of Variation of Velocity.

Similar data concerning the velocities at maxima and minima obtained by previous observers, is given in Table III.

TABLE III - Maxima & Minima of Velocity Curve.

	*1.	2.	3.	4.	5.	6.	7.	8.
Maximum Velocity	Km./Sec. 51.7	41.4	39.7	39.8	31.0	44.3	35.4	26.0
Minimum Velocity	-33.4	-22.9	-29.8	-29.6	-20.0	-28.7	-24.8	-22.0
Range	85.1	64.3	69.5	69.4	51.0	73.0	60.2	48.0
Secondary Maximum	3.3	2.5	.5	8.8	5.0	2.1	6.9	9.0
Secondary Minimum	-4.8	-7.4	-6.4	-.9	-8.0	-6.2	-4.5	1.0
Range	8.1	9.9	6.9	9.7	13.0	8.3	11.4	8.0

The range of velocity is considered more significant in comparison than the maximum or the minimum velocities taken alone. From Table III, it is seen, that the principal range (difference between principal maximum and minimum) is decreasing in passing from Group I to Group II, to the present values. The probable errors of the maximum and minimum velocities are round about 5 Km./Sec. The change therefore may not be altogether real..

* 1 = Ludendorf
2 = Cannon
3 = Jordan
4 = Dustheimer
5 = Schiefer
6 = Group I
7 = Group II
8 = Edinburgh

lengths. Dustheimer is the only observer who took H_ζ in the mean. This tends to make his derived range greater than the true one, because the difference $H - H_\zeta$ is 15 Km. at phase of maximum velocity and +25 Km./Sec. at phase of minimum velocity (See Fig.1, b).

There is no explanation ready for the decreasing range, if it were real, but it has to be taken into consideration in discussing any model of the system. One may recall the last sentence of Jordan's paper (9) - "It seems reasonably certain that the star has extensive chromospheric and absorbing layers whose intensity of action varies regularly with phase, probably because of some tidal action, but beyond this the system is a complete riddle."

The range of the secondary variation does not show any systematic change.

The Blend at 3889.0 A.

The Blend at 3889.0 A consists of H_ζ (3889.05) and HeI (3888.65) separated by .40 A, which is equivalene to 30.8 Km./Sec. Assuming the measured position of the line will be half-way between the components, then the difference $H - H_\zeta$ should be +15.4 Km./Sec. The observed value (Table I - bottom) is +15.2 Km./Sec. Deviations from this mean exist, and are shown graphically in Fig.1, b. The curve has two maxima at phases 40 and 112 days approximately, and two minima at 12 and 70 days. It is to be expected that a maximum of $H - H_\zeta$ would correspond to a weak hydrogen component or a strong helium and the reverse in case of a minimum of $H - H_\zeta$. The hydrogen absorption is strongest at phase 10 days (See below) and to a less extent at at phase 80 days. These phases correspond approximately to the 2 minima of the H - H_ζ curve. The hydrogen absorption is weakest at 35 and 110 days, also in agreement with H - H_ζ curve. The intensity of helium is not actually constant, but it seems that the change in the intensity of the helium component is not strong enough to upset the agreement between the maxima and minima of the H - H_ζ curve and the intensity of hydrogen absorption. As a matter of fact, the description given by Lockyer (13) for the helium intensity is not contradictory. But we cannot be sure that 3888.65 will follow the same intensity changes, as the other helium lines do. This line originates from a metastable level and is expected to be stronger and sharper than other lines, as a consequence of a diluted field of radiation, which is generally assumed to exist in the outer layers of Be atmospheres. In making a setting on a line, the eye is usually attracted by the centre rather than by the wings. This rule may be considered in favour of suggesting a similar contour for two components of comparable intensities for H_ζ blend. This seems to be necessary in order to explain the agreement between the observed wavelength and the arithmetic mean of the two components. Microphotometric tracings made for 10 plates do not show any asymmetry in the line.

The sharp absorption lines in the centre of the Balmer lines.

A notable variation in the spectrum of this star is the extreme change, in number and intensity, of the central sharp absorption. This change is not seen very clearly in the first members, because of the presence of emission components on both sides. For higher members, emission is weak or non-existent, and thus each of these lines appears as a shallow wide absorption, with a very sharp and narrow absorption core in its centre. At some phases this central absorption is not seen at H9 (3834.35A), while at others, it can be seen as far as H24 (3671.45A).

Lockyer (14) was the first to publish observations of these lines, although their variation must have been well known before. Lockyer's observations were eye-estimates. A more accurate method would be a photometric one. Tracings of lines should be made and the intensity and form of central absorption measured. The present material is not standardised, but still it was at first thought possible to use the iron arc spectrum as a calibration standard. This method was developed by Hogg (25), and was used by Lindsay (26) in measuring the Balmer lines. According to them the expected error in the results is of the order of 20 - 50%. The region discussed here (to the violet of 3900A) is full of iron-arc lines, which are so packed together, that they are rarely single. Besides, the plates are usually over-exposed. For these reasons, the error in using the arc lines, is expected to be very large, and thus the method was discarded.

Photometric tracings of some plates were made, and the number of Balmer lines, having sharp absorptions at the centre, was noted. The lines were also counted on the micrometer several times. The two methods gave almost identical results.

TABLE IV.

Quantum number (n_m) of the last Balmer line with sharp central absorption.

Date	Phase (Days)	n_m	D
1936 Dec.22.6.	0	21	.58
1936 Dec.29.8	7	24	.49
1936 Dec.31.8	9	23	.63
1935 Dec.20.9	12	23	.24
1935 Dec.23.9	15	19	.23
1937 Jan. 6.8	15	22	.66
1937 Jan. 7.8	16	24	.69
1937 Jan.16.9	25	18	.61
1938 Feb. 9.8	34	8	.68
1936 Jan.18.8	41	8	.17
1936 Jan.22.8	45	8	.59
1936 Jan.23.8	46	11	.47
1936 Feb. 3.8	57	9.5	.27
1937 Oct.29.8	57	11	.19
1936 Feb.11.8	65	13	.22
1937 Nov.12.9	72	13.5	.39
x1935 Oct.16.6	72	10	-
1937 Nov.20.5	80	15	.15
x1935 Oct.31.7	87	12	-
1937 Dec.15.8	105	8	.47
1936 Dec.12.6	116	8	.83
x1935 Dec. 2.8	120	11	-
1937 Dec.31.8	121	9	.21
x1935 Dec. 4.8	122	12	-
1938 Jan. 5.8	126	14	.27

D is the density of the spectrum measured at 3662 A (See p. 43)

x Single prism plates.

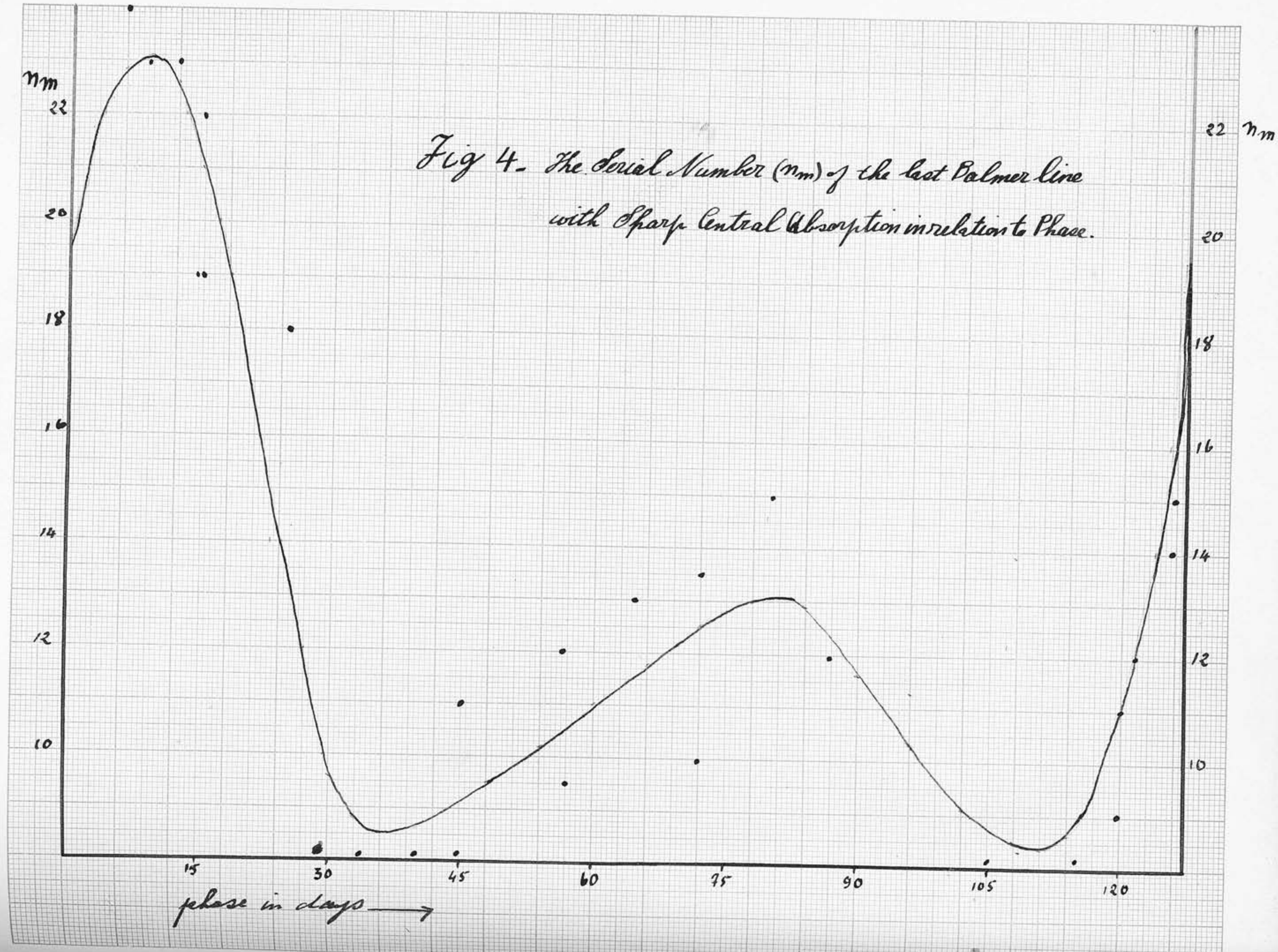


Fig 4. The Serial Number (n_m) of the last Balmer line with Sharp Central Absorption in relation to Phase.

No great accuracy is aimed at in this point, and an error of one or two in the number of lines is to be expected. The writer, intentionally, tended to observe fewer lines in plates which show many lines, and vice versa. This procedure ensures that the observed difference between many-line and few-line plates is smaller than the true one. The serial number n_m of the last line seen having a central sharp absorption core is entered in Table IV and plotted against phase in fig. 4.

Photographic effects of exposure and development are known to be important in determining the number of lines visible in a spectrum. As regards development, the plates were developed under uniform standard conditions, namely with MQ developer for 4 minutes at 65°. As regards exposure, the plates were not equally exposed, and thus the spectra are of different densities. This point will now be discussed. The density D of the spectrum, defined as the log. of ratio of incident to transmitted light,* is also given in Table IV. D was measured at a point corresponding to H30 (3662 A). The values of D are grouped in three groups, viz.

(I) phases 125-30 days, for which the values of n_m on the curve are high;

(II) phases 30-50 and 100-116, for which the curve gives a low value of n;

and (III) ~~intermittent~~ intermidiate phases, then we get :-

Group	Phase	Mean n	Mean D
I	125 - 30	21.2	.47
II	(30 - 50) (100- 116)	8.6	.51
III	Other phases	12.1	.30

* Strictly, D is the log of the ratio of the microphotometer beam passing through a clear part of the plate to that passing through the image.

The mean D for group a (large n) is comparable to, but actually slightly less than, the mean D for group II (small n); and the mean D for the intermediate group is smaller. This seems to show that the observed change in the number of lines cannot be due to exposure. In fact, on plotting D against n, it will be seen that the correlation is an extremely bad one. Thus it seems reasonable to conclude that the photographic and observational errors in determining n_m, are relatively small, and would not affect the results.

The outstanding feature in curve 4 is the increase in n_m from 9 at phase 115 days to 23 at phase 10 days, and its subsequent return to a value of 9, 20 days later. The first physical process that suggests itself is a change in pressure.

It is well known that the number of absorption lines in the Balmer series is determined by several factors, the most important of which is the electron pressure. Higher ionic concentration affects the outermost orbits so as to merge the lines near the series limit with the continuous. Recently, Inglis and Teller (27) derived a formula connecting the ionic concentration N with the maximum quantum number n_m, which is the effective quantum number of the line with which the series terminates. The formula $N = .027\ a_o^{-3}\ n_m^{-7.5}$, where a_o is the Bohr radius, can be applied to any one-electron spectra. Inglis and Teller have shown that for $T > 10^5/n_m$, $N = 2\ Ne$, where Ne is the electron concentration. Mohler (28) verified the formula experimentally for the emission spectrum of caesium.

By introducing the relation Pe = KTNe where Pe is the electron pressure, Mohler obtained the working formula -

$$\log Pe = 1.19 + \log T - 7.5 \log n_m .$$

The physical significance of Pe, derived from this formula, requires consideration. At a given wavelength the light comes from an effective depth in the stellar atmosphere which depends on the opacity of the atmosphere at that wavelength. Values of Pe, obtained from the number of lines in the Balmer series pertain to the electron pressure at the photospheric layer corresponding to the Balmer limit.

The observed change in n_m for the series of sharp absorption may be due either to

(a) a real change in the apparent limit of the series, due to a change in pressure;

or to (b) a moving atmosphere of hydrogen, capable of producing sharp absorbing lines, coming in the line of sight during the corresponding phase.

Assuming case (a) above, we see (using Mohler's formula given above) that a change in n_m from 9 to 24 corresponds to a decrease in electron pressure of 1150 fold. Such a change in pressure of the order 10^3, in an interval of 20 days, is considered rather improbable.

We have next to consider the possibility (b). In this case we assume two atmospheres of widely different electron pressures, associated with two components of a binary system. The atmosphere of the primary corresponds to an ordinary Bo spectra and may be in rotation. It gives rise to the wide shallow absorption lines. The secondary star has a very rarefied and extended atmosphere and is likely to be of low luminosity. Sharp central absorptions originate in the atmosphere of the secondary, when it ~~is eclipsed by~~ eclipses the primary during the phase 115-35 days.[x] The duration of the eclipse (roughly measured by the interval between the phases corresponding to a value of $n_m = 16$ on ascending and descending branches) amounts to 24 days, compared with 126.626 days, which is the complete period. It may be assumed from this that the dimensions of the stars are not very small compared with those of the orbit.

Had this eclipsing hypothesis been true, we should have observed the intensity of central absorption of $H\gamma$ or $H\beta$ having a curve of change with phase similar to that in Fig. 4. This is not the case (See reference 17, p.571). Observations of absorption in $H\gamma$ and $H\beta$ are complicated by the presence and variation of emission, and therefore, the above objection may not be a serious one.

Photometric observations of the star were carried out by Guthnick (29), Güssow (30) and recently by Roach† (31).

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[x] Sharp absorption lines may originate also in the atmosphere of the primary, when it ~~is eclipsed by~~ eclipses the secondary. But they will not extend far in the ultra violet because of the relatively high pressure prevailing. This may explain the secondary maximum at phase 80 days in Fig. 4.

† Roach observed the star from phase 35 to phase 76. He did not find any indication of an eclipse in the interval. The current discussion makes it likely that an eclipse, if it ever happens, should be expected at about phase 10 days (corresponding to the maximum in Fig.4) and not during the secondary variation, where Roach was searching for it. For further discussion of Roach's results, see p.59.

Several small fluctuations have been observed, but there is nothing similar to a regular eclipse. This may be reconciled with the eclipsing hypothesis by assuming the secondary atmosphere to be very rarefied and extended, and thus of low opacity.

Further study of the system seems to be most promising along the following lines :-

(1) Accurate photometric study of Balmer-emission and its variation with phase;

(2) Measurement of the variation of the shallow absorption lines both in position and intensity. These lines have always evaded study;

(3) Observing the helium lines originating from metastable levels in the ultra-violet region, especially 3187 ($2^3S - 4^3P$), which is expected to be the strongest line in the whole spectrum. This determines the dilution factor.

(4) Re-observing the secondary variation peculiarities. These furnish a good idea about the stratification and extent of the absorbing layers.

Not before these points, as well as many others, are cleared, can any model of the system be proposed. All that can be said now is that the system is very likely to be a spectroscopic binary, to which a suffix (p) must be added.

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PART III.

Line Intensitites in Some Be-Stars.

Abstract.

Line intensities in seven plates of ϕ Per., covering about 30 days of the star's period were measured. A clear correlation is found between emission line intensity and the colour of the star as given by Roach. The widths of emission lines in some Be-stars are compared with the value computed from Curtiss' formula. The possibility of measuring the hydrogen-decrement in Be-stars is discussed and the previous observations criticised.

Equivalent widths of helium and hydrogen lines in κ Cas. and β Ceph. are also given. H α is definitely variable in κ Cas., while Hβ is apparently constant. The most probable turbulent velocity is obtained for the two stars.

LINE INTENSITIES IN SOME Be-STARS.

Introduction.

In the present section, the total intensities of emission and absorption lines in some Be-stars are given. The original purpose of the measurement was to provide accurate total intensities for some Be variables, especially ϕ Per. But owing to certain ~~some~~ reasons the material necessary did not become available, and therefore the finished observations are given here without a complete discussion.

The work done on the photometry of emission lines in Be-stars is relatively small. It is true that the available theories are not refined enough to stand any quantative test, but this does not belittle the importance of having accurate measures of the intensities of these lines. Valuable results have already been obtained from the qualitative data and rough estimates of unstandardised material; and the calibrated plates should prove more valuable.

The presence of absorption and emission components near each other introduces additional photographic and photometric errors. In many cases the emission is too strong relative to the adjacent continuous, and therefore choice has to be made between a measureable continuous and an over-exposed line, or a measureable line with an under-exposed continuous.

Another difficulty in measuring these lines occurs in choosing the level from which the total intensities are measured. A typical line consists of a double emission superimposed upon a wide absorption, and itself bisected by a central sharp absorption. In the line $H\alpha$, the emission is usually strong enough to mask a considerable part of the absorption, and in higher members of the Balmer series, the emission is too weak to interfere seriously with the absorption. Therefore for $H\alpha$ the emission can be measured with some certainty, while for higher members the absorption will be thus measured. It is when emission and absorption are of comparable intensities that the difficulties are very serious.

Some writers tried to overcome these difficulties by using some approximation, either by extending the absorption contour underneath the emission, or by assuming a total intensity of the absorption line. Reference will be made to this point later.

ϕ Persei.

The radial velocity of the star is variable in a period of 126.626 days. Every Balmer line consists of 3 components, wide absorption, double emission and sharp central absorption. The 2 emission components (V and R) vary in intensity relative to each other in the same period as the radial velocity, and the total intensity of emission as a whole is suspected of being variable in half that period.

The Material & Observation.

The material consists of 7 spectra taken with the two-prism spectrograph. They are calibrated in a multiple-slit spectrograph and measured in a Moll-type micro-photometer arranged for direct readings of the galvanometer deflection. (for

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more particulars see p.85, ff). Full data are given in Table I. 85

TABLE I.

No.	Plate	Data	Phase	Lines measured
1	63-38	Oct.25.8	38.6	$H\alpha$ - $H\beta$ - 4472 - $H\gamma$
2	65-38	Nov.15.9	59.7	$H\alpha$ - $H\beta$ - 4472 - $H\gamma$
3	45-39	Nov.23.9	53.7	6678 - 5876 $H\beta$ - 4472 - $H\gamma$
4	45-39	Nov.23.9	53.7	$H\alpha$ - $H\beta$ - 4472 - $H\gamma$
5	50-39	Dec. 2.9	62.7	$H\alpha$ - 5876 - $H\beta$ - 4472 - $H\gamma$
6	53-39	Dec. 5.9	65.7	6678 - $H\alpha$ - 5876 - $H\beta$ 4472 - $H\gamma$ - $H\delta$
7	55-39	Dec. 6.9	66.7	5876 - $H\beta$- 4472 - $H\gamma$

The phase is computed in days using Dustheimer's epoch and period (p.32). 32

The plates are grouped according to phase in 4 groups:-

Group I	No. 1	Phase	38.6 days
Group II	" 3 & 4	"	53.7 "
Group III	" 2 & 5	"	61.2 "
& Group IV	" 6 & 7	"	66.2 "

A separate reduction curve was constructed for every line measured on each plate. Enough distance on both sides of the lines was taken to judge the continuous. Owing to the presence of faint wide emission lines of FeII, the determination of the continuous was difficult, especially near $H\gamma$ and 4472, where many emission lines are found. Regions free from such lines were chosen, using the list of absorption Fe II lines in α Cyg. (1) and emission lines of Fe II in γ Cas. (2)

For each line the continuous was inserted in the first place by drawing a straight line (using eye estimation) on the plot of Δ (see p.86) against wave-length. The values of were then converted into light throughout the line, the adopted continuous being taken as 100.

A subsequent revision was made by meaning the whole series of seven plates for the points selected as defining the continuous. The final means for these points were different from 100 and factors, varying with the wave-length were deduced so as to make the continuous equal to 100 on both sides of the line. These factors were then used for each plate separately. The total intensities are expressed in units of the energy within 1 A of the interpolated continuous at the wave-length of the line and refer to the area between the contour and the interpolated continuous.

Hα: The dispersion at Hα is 133.4 A/mm. The line appears as single emission, probably with some weak absorption. The small dispersion does not permit any duplicity to be seen, neither is there any asymmetry. The measures are given in Table II.

TABLE II.

Intensity & Width of Hα in ϕ Per.

Group	Phase	Total Emission	Deviation	Width
I	38.6	54.0	-11.1	19.3 A
II	53.7	67.3	+ 2.7	15.9 A
III	61.2	70.3	+ 5.2	16.9 A
IV	66.2	68.7	+ 3.6	13.7 A

The widths in Angstroms given in the last column are the distances between the two points on both sides of the centre at which the intensity equals half its value at the centre of the line. The deviations from the mean total intensity are considered significant for Group I. From other evidence (measures of Hβ) it is likely that the deviation is real for other groups.

Hβ: Hβ is not purely emission; the wide absorption is seen on some plates; the central absorption divides the emission clearly into two components. Owing to the

$H\beta$ (Contd.):

difficulties mentioned before, the exact amount of emission or absorption in the lines cannot be found, and therefore the results will be recorded here in the following way. The line is divided into three parts, the first extending 5 A on both sides of the centre, and the second and third parts extend from 5 → 15 A on either side. The central part contains the main portion of the emission, while the other two parts indicate the emission or absorption in the wings. The choice of these limits is not altogether arbitrary; the contour of the line in different plates suggests it. Only a very small fraction of emission or absorption is found outside the interval +5 → -5 A.

The width of $H\beta$ is defined as the distance in Angstroms between two points A and B. A is the point on the red side of the red component, at which the intensity is half the maximum intensity of the red component. B is similarly the point, on the violet side of the violet component, at which the intensity is half the maximum intensity of the violet component. The difference between the red and violet components of emission cannot be disentangled from the photographic effects, especially because the two components are of comparable intensity and not well separated. The dispersion at $H\beta$ is 46 A/mm. The results are given in Table III.

TABLE III.

Intensity & Width of $H\beta$.

Group	$+15 \rightarrow +5$	$+5 \rightarrow -5$	$-5 \rightarrow -15$	Net emission	Width
I	+.26	+3.68	-.16	3.78	7.1
II	+ 00	+5.16	+.15	5.31	7.7
III	+.06	+5.65	+.11	5.82	6.5
IV	+.16	+4.69	+.18	5.03	7.3

+ is emission and - absorption

$H\gamma$: The dispersion of $H\gamma$ is 28 A/mm. The central and wide absorptions are stronger and the emission components are weaker, compared with $H\beta$. The contour is divided into 7 parts as follows :-

1 & 7	from	$\mp 15 \rightarrow \mp 8.7$ A
2 & 6	"	$\mp 8.7 \rightarrow \mp 3.0$ A
3 & 5	"	$\mp 3.0 \rightarrow \mp 0.9$ A
4	"	$+ .9 \rightarrow - .9$

The central part (4) is usually filled with the sharp central absorption; (3) and (5) correspond to the violet and red emission components respectively; while the other 4 parts define the wings of the line. The width of the line is measured in the same way as for $H\beta$. The next table shows the measures; the numbers given at the top of the columns refer to the parts explained above.

TABLE IV: Intensity & Width of $H\gamma$.

Group	1	2	3	4	5	6	7	Width
I	-.17	-.18	+.15	-.15	+.11	-.27	+.07	5.4
II	00	-.27	+.23	-.42	+.14	-.42	+.04	4.6
III	+.12	-.46	+.20	-.39	+.25	-.40	+.13	4.9
IV	-.03	-.44	+.20	-.36	+.21	-.26	+.19	4.6

The second decimal given in the last two tables are not of much significance; they are entered for sake of completeness.

Discussion.

1: The accuracy of the measurements:- The main purpose of measuring ϕ Per. plates is to note any variation in the intensity. Before doing so, the errors involved have to be determined. Five plates of O Cas. were measured for this purpose. This star has a bright single $H\alpha$, weak dark $H\beta$ and a normal $H\gamma$ line. The means of the plates were computed, and deviations of each plate from this mean for every line were plotted against wave-length. Thus for every line, $H\beta$ say, five curves are obtained, showing the deviations (using .01 of the continuous as a unit) from the mean at each wave-length on both sides of the centre of the line. Deviations were largest for $H\alpha$ and smallest for $H\gamma$. The mean error in making a single setting at $H\alpha$ is 3.9% of the continuous

and for $H\beta$ is 3.3% " " "

and for $H\gamma$ is 2.5% " " "

and " 4472 is 1.9% " " "

Thus on the average a single setting at $H\gamma$ deviates from the mean of 5 measures by .025 of the continuous. This rough method of estimating the errors is empirical in the sense that all the different sources of errors are combined together.

The effect of these deviations on the measured intensity depends on the number of points at which the contour is determined. As an example, for $H\gamma$, the mean deviation per setting is

2.5%; in measuring the line 30 settings were made, the interval between which is .3A, and another 30 points with an interval of .75A. The maximum error in measuring the whole line is = 30 x .025 x .3 + 30 x .025 x .75 = .79A. The total intensity of the line is 4.58A. Thus the maximum error is $\frac{.79}{4.58} = 17\%$. The actual error must be less than that, since not all deviations will have the same sign, therefore the error in measuring the total intensity of the line may be estimated to be 10%.

In the case of Hβ, the line is situated in an unfavourable region on the plate, and for Hα the small dispersion combined with the large grain produces larger errors. It is concluded that errors in the total intensity range from 10$\rightarrow$20%. For limited parts of a line, the mean deviation per setting given above were found more useful. The material is not sufficient to obtain probable errors of the total intensities in the ordinary way.

<u>2. Variations in ϕ Per. lines</u>:- The method used here is the same as explained above, i.e. deviations from the mean are plotted for each group against wave-length. The plates measured cover the phases from 39 - 66 days, which correspond to the secondary variation (see previous section p.34). The first group is shortly before the secondary minimum velocity (45 days), and group III corresponds to the secondary maximum velocity (60 days).

Fig. 1(a)

Fig 1(b)

Fig 1(c)

Fig 1(d)

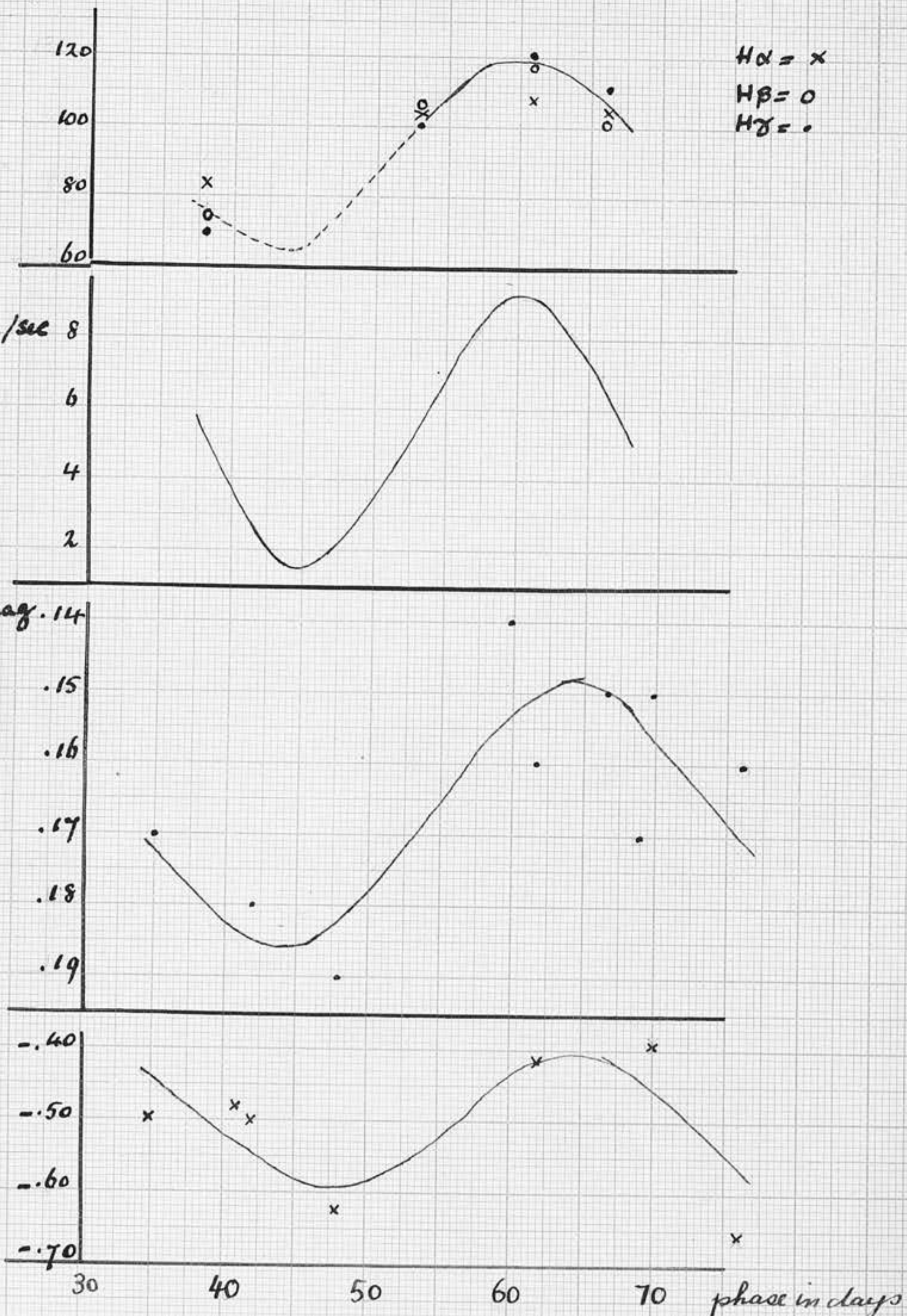

Fig 1(a) = The total emission in the Balmer lines in φ Per - at different phases referred to the mean of all measurements (= 100)

Fig 1(b) = Radial velocity of the central hydrogen absorption at the corresponding phases.

Fig 1(c) = Δmag = φ And - φ Per as measured by Roach.

Fig 1(d) = Colour index of φ Per " " "

a: Variation of the intensity:

The intensity of emission in $H\alpha$, $H\beta$, and $H\gamma$ were referred to their mean. It is found that the intensity of emission increases from Group I onwards, reaches a maximum at Group III and then decreases. The result is shown graphically in Fig.I(a). The radial velocity of the central hydrogen absorption is taken from Fig.I (a) (p.34) and given for comparison in Fig.I (b). A close correlation is suggested between the two curves, but the observations are not numerous enough to warrant a definite conclusion.

An increase in the emission may result from :-

(1) a decrease in the absorption, and thus unmasking more emission;

(2) a decrease in the intensity of the continuous;

or (3) a real increase in the number of atoms producing the line.

Recent photometric measures of the star were done by Roach (3) using a photo-electric photometer. Roach concluded that his observations do not indicate any eclipse during the abnormal phase (i.e. during the secondary variation, discussed here), although the star appears to vary in an irregular manner. Magnitude observations published by Roach are shown in Fig.I(c); the ordinates are Δ mag. = ϕ And. - ϕ Per. ϕ And. (B8) is fainter than ϕ Per and thus the larger the difference Δ mag, the brighter is ϕ Per. The changes are very small, do not exceed .05 mag., but the distribution of the points suggests the curve drawn. Roach's colour index observations are given in Fig.I (d). A minus colour index indicates a greater response through the blue filter than through the yellow.

A comparison of the four curves indicates that at phase 60 (max-velocity) the emission lines are stronger, and the star is fainter and redder than at phase 45 (minimum velocity). The similarity between the two curves I (a) and I (d) shows a dependence of the colour on the intensity of emission, but this does not exclude the possibility that the distribution of light in the continuous spectrum is changing too. The fluctuations of magnitude in this short interval may be of importance in defining the star's model.

Concerning the change in absorption during the interval, the central sharp absorption increases from phase 35 d $\rightarrow$ phase 70; this is seen at once from Fig.4 (p.43), and can be detected from the intensities in column 4, in the last table. The wide absorption cannot be measured accurately, but the deviation-curves do not indicate any definite change. Therefore the possibility (1) above may be ruled out.

(b): The Width of the Emission Lines.

The widths of emission lines, defined by the diatance between the steepest outer gradients of intensity, show, according to R. H. Curtiss (4), a pronounced proportionality to wave-length. Curtiss obtained an empirical formula, giving the width of the emission line ($A\lambda$) at wave-length (λ), in terms of the width (W) of H β in the same spectrum.

The formula is $A\lambda = 6.28 \times 10^{-4} (\lambda - 3270)(W - 2.61) + 2.61.$

This relation holds reasonably well in the blue region, but as far as can be ascertained, it has not been tested in the red region. The proportionality with wave-length is one of the strong points in favour of Struve's rotational hypothesis of Be-stars.

TABLE V - Width of Emission Lines.

Star	Hα			Hγ		
	O	C	O - C	O	C	O - C
γ Cas. (1)	8.0	6.8	+1.2	3.3	3.9	- .6
K Cas. (5)	9.3	- -	- -	- -	- -	- -
O Cas. (5)	8.2	8.8	-.6	- -	- -	- -
ψ Per. (2)	9.3	9.6	-.3	4.2	4.8	-.6
υ Cyg. (1)	6.7	6.8	-.1	2.6	3.9	-1.3
φ Per. C	- -	- -	- -	5.6	5.5	+.1
φ Per. D	- -	- -	- -	5.6	5.5	+.1
φ Per. E	16.5	9.7	+6.8	4.9	5.6	-.7

The widths of Hα, Hβ, and Hγ were measured in some stars beside φ Per. The widths are given by Table V in A-units. The number of plates measured is written after the name of the star in the first column. Thus for Hα and Hγ the observed (O) width, the computed width (C) from Curtiss' formula, and (O - C) are given. For φ Per., the measures by Curtiss and by Dustheimer (5) are tabulated for comparison with the Edinburgh measures (bottom row). The formula is derived from widths measured visually under the micrometer, but the present widths are obtained from photometric contours. As regards φ Per., the values given by Curtiss and Dustheimer are the means for the whole cycle, while Edinburgh widths cover only a limited part.

The residuals (O - C) are not altogether large considering the empirical nature of the formula and the accuracy of the measures.

(O - C) for H α in ϕ Per. is exceptionally large and may be real. (O - C) for H γ in the same star is reasonable; thus there can be no serious error in the width of Hβ.

c: Relative Intensities of the Balmer Lines:-

Comparison between intensities of lines at different wave-lengths is a difficult and important problem. First the total intensity is measured relative to the neighbouring continuous, and next, the distribution of energy within the continuous. The black-body distribution is only a rough approximation, although the deviations from it in the visible region is generally small even for some stages of novae (6). The accurate method for deriving relative intensities is by comparing the star's continuous with another standardised source, which may be a standard lamp or star.

In the case of Be-type stars in general and ϕ Per. group in particular, the first step in determining the total intensity relative to the adjacent spectrum, is in itself very difficult, because of the complications explained before (p. 51).

Therefore, accurate relative intensities for such stars are almost out of question, for the time being at least. In spite of this conclusion, many observers have measured relative intensities of emission lines in Be-stars with "some" approximations, and reached important results.

Struve & Schwede (7) used the two following approximations:-

(1) Neglect the central absorption and extend the outer absorption under neath the emission, so as to form a contour similar to the contours of the same line in absorption-line stars of

the same rotation and spectral subdivision.

(2) For defining the energy of the continuous, assume black body radiation and the ionisation scale of temperatures for different subdivisions.

Karpov used much better approximations :- (8)

(1) He neglected the central absorption in the stars. For the outer absorption he compared the higher members of the Balmer series in the emission line star under investigation, with the corresponding lines in some purely absorption-star; then he finds for every Be-star an absorption-line star having equal total intensities in the higher members of the Balmer series. Then he assumed this equality to hold for $H\alpha$, $H\beta$ and $H\gamma$, and by measuring the apparent absorption and subtracting it from the assumed total absorption, the total intensity of emission is found.

(2) Karpov derived colour temperatures for his stars by measuring the continuous at three wave-lengths and using γ Peg. as a standard; and then he assumed black body radiation corresponding to the derived colour-temperature.

The relative intensities are then compared (usually relative to $H\beta$ as unit) with the theoretical intensities formed through ionisation and recombination or, with the values measured in the planetary nebulae by H. H. Plaskett and Berman. Invariably it is found that the agreement is reasonably good and entirely unexpected. This may be due, among other things, to the insensitivity of the Balmer decrement as a measure of temperature (Menzel 9).

In this connection a paper by Mohler (10) may be mentioned, in which he applied Zanstra's theory to determine the temperature of Be-stars, treating them as nuclei of planetary nebulae. During this work, the total emission of the Balmer lines were measured. Mohler's list contains many stars such as ϕ Per. and ψ Per., which are known to have strong central absorption lines. There is no reference in Mohler's paper about the method of approximation used in obtaining his measures.

It is possible to predict Balmer line intensity ratios from the velocity distribution of free electrons (defining the electron temperature), the capture coefficient and the Einstein transition probabilities. The theoretical Balmer decrement was computed by Cilliè (11) and by Menzel and his co-workers (12). Baker and Menzel (13) found that "measures of the Balmer decrement are unsuited to determinations of the temperatures of the electron gas .... The observed Balmer decrement can be used to indicate the physical nature of the excitation."

On the observational side, nebular line intensities have been determined by Wright (14), H. H. Plaskett (15), Berman (16) and Page (17). Deviations from predicted values were found, and partly explained by self-reversal (18), and selective interstellar absorption (16).

Table VI gives a summary of the theoretical and observed values. All quantities are expressed in terms of H_β as an arbitrary unit of intensity. Theoretical ratios are those calculated by Cilliè (11) and by Baker and Menzel for T = 20'000. The observed values, as given by Berman, are means of 17 nebulae, for which the nuclear temperatures are lowest. The number of

Be-stars included in each mean are as follows :-

Struve & Schwede	14
Mohler	11 ($H\delta$, 7 only)
Karpov	13

TABLE VI.

	$H\alpha$	$H\gamma$	$H\delta$
Cilliè	2.88	.48	.27
Menzel & Baker	2.59	.50	.30
Berman (Nebulae)	2.77	.50	.26
Be-Star Struve & Schwede	- -	.39	.27
Mohler	2.07	.64	.49
Karpov	2.88	.45	.30

In comparing Be-stars with nebulae and theoretical results it is noticed that :-

(1) the theoretical results are computed for recombination while fluorescence may be of importance, especially in late Be-stars;

(2) the assumption that the physical conditions in the outer atmosphere of Be-stars are comparable with the planetary nebulae, may introduce serious errors, especially as regards self-absorption.

These two factors tend to give steeper Balmer decrements in Be-type stars. Table VI shows that such an effect is not present.

Still, it may be argued that the measures for Be-stars are self-consistent, i.e. have inner accuracy as a result of the approximation method employed. To test such a possibility, although it is very unlikely, Table VII is constructed. It gives the observed ratios for each spectral subdivision. The number of stars in each case is introduced after the ratio. The means, for every observer separately, were plotted against spectral type. The scatter is found to be extremely large, and no tendency towards steeper decrements for later types could be traced.

Merrill (19) has pointed out that, "the rapid decrease in the intensity of the emission in passing to the more refrangible members of the Balmer series, as well as the increase in the absorption, is associated with the later subdivisions of class B. In class Bo, for example, it is usually more gradual." Therefore the steepening of Balmer decrement for later sub-types is expected from theoretical, as well as observational, evidence. The published values of the Balmer decrement, as shown in Table VII, do not confirm this conclusion. Thus, either the change with spectral type does not exist, or the measures are too rough to show it. The latter conclusion is probably nearer the truth.

TABLE VII.

Balmer Decrement in Be-type Stars.

Type	Observer	Hα		Hγ		Hδ	
Bo	S & Sc	- -	-	4.0	2	3.1	2
--	M	34.3	2	6.2	2	4.5	2
--	K	26.6	1	5.0	1	3.2	1
B1	K	26.4	1	5.0	1	- -	-
B2	M	47.7	1	5.4	1	1.2	1
	K	31.5	1	4.2	1	3.3	1
B3	S & S	- -	-	3.7	10	2.2	2
	M	38.2	5	5.1	5	3.9	3
	K	19.3	3	4.3	3	3.1	3
B5	S & S	- -	-	4.9	2	4.6	2
	M	43.0	3	4.6	2	3.0	1
	K	29.5	4	4.5	3	2.9	3

Note: The measures given by Mohler are the ratios A_α/A_β, A_γ/A_β ...
where A_ν are defined by the relation

$$A_\nu = \int_{\lambda_1}^{\lambda_2} I_\lambda \cdot d\lambda \Big/ I_s \cdot \lambda$$

where I_s refers to the interpolated continuous. the ratios are very nearly the same as the intensity ratios except for Hα

~~and are very nearly the same as the intensity ratios, except for $H\alpha$.~~

To illustrate the effect of the method of approximation on the derived decrement, the total intensities in ϕ Per. may be used.

(1) If a dish-shaped contour is assumed for the wide absorption, and the emission is measured above it, the mean relative intensities for the measured 7 plates of ϕ Per. are found

$$H\alpha : H\beta : H\gamma :: 5.05 : 1.00 : .22.$$

(2) If the total absorption in ϕ Per. is assumed to be the same as in ϵ Ori., then the following ratios are obtained :-

$$H\alpha : H\beta : H\gamma :: 3.50 : 1.00 : .45.$$

The total absorptions for $H\alpha$, $H\beta$ & $H\gamma$ measured by Günther (20) were used. If, however, other values or other stars are used, different ratios will be obtained.

From the previous discussion, it is concluded that, the determination of the Balmer decrement in Be-stars is very uncertain, both from theoretical and observational reasons. The published results are considered rather premature.

κ CASSIOPEIAE

R.A. $0^h\ 27.3^m$ Mag. 4.2
Decl. $62^\circ\ 23'$ Type BoSE

Hα was discovered in emission in this star by Wilson (21). Hβ, Hγ and Hδ appeared as rather narrow absorption lines. They had no trace of emission but they were weaker than in many other Bo-stars. Struve (22) had previously called attention to the fact that the absorption lines in this star changed considerably between September 1908 and July 1929. Of special interest is the change in Hβ reported by Struve between September 8 and October 5, 1908. Wilson did not observe any change on his two plates (20 days apart).

Merrill (23) found the star to be a spectroscopic binary, no orbit has so far been determined, and the velocity range is probably not large (24). According to Hunter and Martin (25), κ Cas. is one of the stars which are abnormally reddened on the basis of spectroscopic distances, but are not particularly so if K-line distances are accepted.

<u>Material & Measures:</u> Four standardised double prism plates were measured in the usual way. Data concerning the material is given in Table VIII.

TABLE VIII.

No.	Plate	Date	Exposure Time	Calibration	Slit
1	62/38	1938 Oct.25.9	30^m	2^m *	075^{mm} †
2	64/38	1938 Nov.15.9	40	2 *	"
3	39/39	1939 Nov.15.9	45	45	"
4	43/39 x	1939 Nov.23.9	40	40	"

x An iron arc comparison was photographed on this plate.

* A correction for short calibration exposure was applied. This was obtained from long and short exposures on a lamp using the calibration spectrograph.

† The slits are the actual slit - settings and should be multiplied by the ratio of the focal lengths of camera and collimator (,625) to give equivalent slits.

$H\alpha$ was measured on two additional plates: $\frac{2}{39}$ - taken on Jan. 9.8, 1939 and 35/39 taken on Nov.6.9, 1939.

The equivalent widths of the lines measured are given in the next table. The unit used is 1A; 1, 2, 3 and 4 refer to the plates given in the previous table.

TABLE IX.

Equivalent Widths of Lines in κ Cas.

Atom	Line	1	2	3	4	Mean	Notes
HeI	6678	2.07	1.53	1.37	1.06	1.51	
$H\alpha$	6563	3.09	1.71	3.26	3.36	2.99	1
NaI	5896	.38	.32	.30	.47	.37	2
NaI	5889	.54	.55	.44	.51	.51	2
HeI	5876	.87	.88	1.10	1.00	.96	
HeI	4922	.84	.78	.62	.86	.78	6
$H\beta$	4863	1.15	1.12	1.26	1.06	1.15	3
HeI	4472	.70	.73	.99	.83	.84	
HeI	4388	.55	.29	.38	.35	.39	
$H\gamma$	4340	1.44	1.34	1.59	1.41	1.45	
HeI	4143	- -	.40	.18	.39	.32	4
$H\delta$	4101	- -	.71	.73	.83	.76	5

NOTES: (1) The line is in emission, single and symmetric, in plates 3 and 4; however the red side may be stronger, but the dispersion is too small to ascertain that. Plate 2 is abnormal, since the intensity of the line is almost half the mean of the other three plates. This indicates that $H\alpha$ is variable. The interval between plates 1 and 2 is 21 days. The total intensity of $H\alpha$ on plate $\frac{3}{39}$ (55 days after plate 2) was found to be

3.47 A, and on plate 35/39, 3.52 A. The mean intensity of $H\alpha$ in five plates (excluding plate 2) is 3.34 A. Plate 2 was remeasured, giving a value of 1.78 A.

Three quartz prism plates of this star were examined visually -

On plate	12/37	1937 Jan.14	$H\alpha$ is faint emission.
" "	15/37	1937 Jan.16	$H\alpha$ is a strong emission compared with the previous plate, taken two days before.
" "	25/37	1937 Feb. 6	$H\alpha$ is weaker than on the previous plate, but stronger than in the first.

On these three plates $H\beta$ is almost the same. Thus it seems that $H\alpha$ varies in intensity, but the period of variation cannot be defined, except from a continuous series of observations.

(2) It is likely that these two lines are of interstellar origin. K of CaII has a radial velocity different from that of the star. The ratio of intensity $\frac{D2}{D1} = \frac{.51}{.37} = 1.38$ compared with ratio in the sun's disc = 1.50

Ratio in the sun-spots = 1.40

theoretical square root = 1.41

(3) $H\beta$ The four measures are very consistent, considering the expected errors of measurement. There is no asymmetry in the line. Here also the present material agrees with Wilson's findings. It must be concluded that the variation which affects $H\alpha$ so strongly, has not any appreciable effect on $H\beta$, either because the emission in $H\beta$ is too faint, or because of some change in $H\beta$-absorption. The last possibility is faintly suggested by the measures of $H\zeta$ and $H\gamma$ given in the table.

(4) This line is not well defined.

(5) The focus at $H\zeta$ is not perfect; the measures are given here for completeness.

(6) This line is found in a region of bad sensitivity of the plates, and must be considered relatively uncertain.

The Helium Lines.

In the next section the relative intensities of the SiIII lines are discussed and the conclusion is reached that the damping constant may be more than 440 times the classical value. The recent determination of the curve of growth for helium lines in early-type stars by Goldberg (26) is referred to. Goldberg computed the oscillator strengths of the diffused singlet and triplet series of helium (27), and used the equivalent widths of nine helium lines measured by E. G. Williams (28). The two lines 6678 and 5876 were not measured by Williams. It is interesting to see from the present measures how far they will fit Goldberg's results, especially his finding that all the observable lines of helium fall either on the Doppler or the flat portion of the curve of growth. These two lines have also the important property of being less affected by Stark effect (29); and it was suggested (28) that they would be very suitable for determining the abundance of helium.

Following Goldberg, the Doppler portion of the curve of growth is given by the equation $\frac{W}{\lambda} \propto \sqrt{\pi}\,\frac{v_0}{c}\,X_0$

and the flat intermediate part by $\frac{W}{\lambda} \propto \frac{2v_0}{c}\left(\log X_0/.434\right)^{\frac{1}{2}}$

where W = equivalent width of the line

λ = wave-length of the line centre

v_0 = most probable kinetic velocity of the atoms

X_0 = optical depth at the centre of the line.

$$X_0 = \frac{N_a}{b(T)} \cdot e^{-\chi_{J}'/KT} \cdot \frac{1}{3\sqrt{\pi}R} \cdot \frac{\pi\epsilon^2}{mc} \cdot \frac{c}{v_0} \cdot S$$

where ~~and~~ N_a is the total number of atoms of the element in the appropriate state of excitation per square centimetre above the photosphere;

$b(T)$ is the partition function;

χ_{J}' is the lower excitation potential;

and S = the strength of the line in atomic units. Introducing the numerical values of the constants for helium at 20'000 Goldberg reduced the above two equations to

$$\log \frac{W}{\lambda} = \log X_0 - 4.27 .$$

$$\text{and } \log \frac{W}{\lambda} = \frac{1}{2} \log(\log X_0) - 4.04 .$$

and on plotting these two curves and joining them asymptotically, the theoretical curve is obtained. (Fig.I - Goldberg's paper).

Assuming that the main opacity in high-temperature stars is produced by the ionisation of hydrogen, then the opacity should vary nearly as λ^3, and(Na)will be consequently variable. Taking a standard wave-length λ' (=5'000A), the value of(Na)will be approximately given by $Na = Na'(\lambda'/\lambda)^3$

Introducing this expression, X_0 will be given by

$$\log X_0 = \Delta' + \log X_0'$$

where $\log X_0' = \log S + 3 \log \lambda'/\lambda$ and will be variable from line to line

and $\Delta' = -11.16 + \log \frac{Na'}{b(T)} - \frac{5040}{T}\chi_{J}' + \frac{1}{2}\log T_0$, Δ' can be considered as constant if the differences in excitation potentials between different diffuse lines of helium are neglected.

The effect of temperature variation, turbulence and Stark effect on the Doppler and intermediate branches, may be accounted for, on the average, by introducing a new velocity parameter v' such that $\log v' = \log v_0 + V$

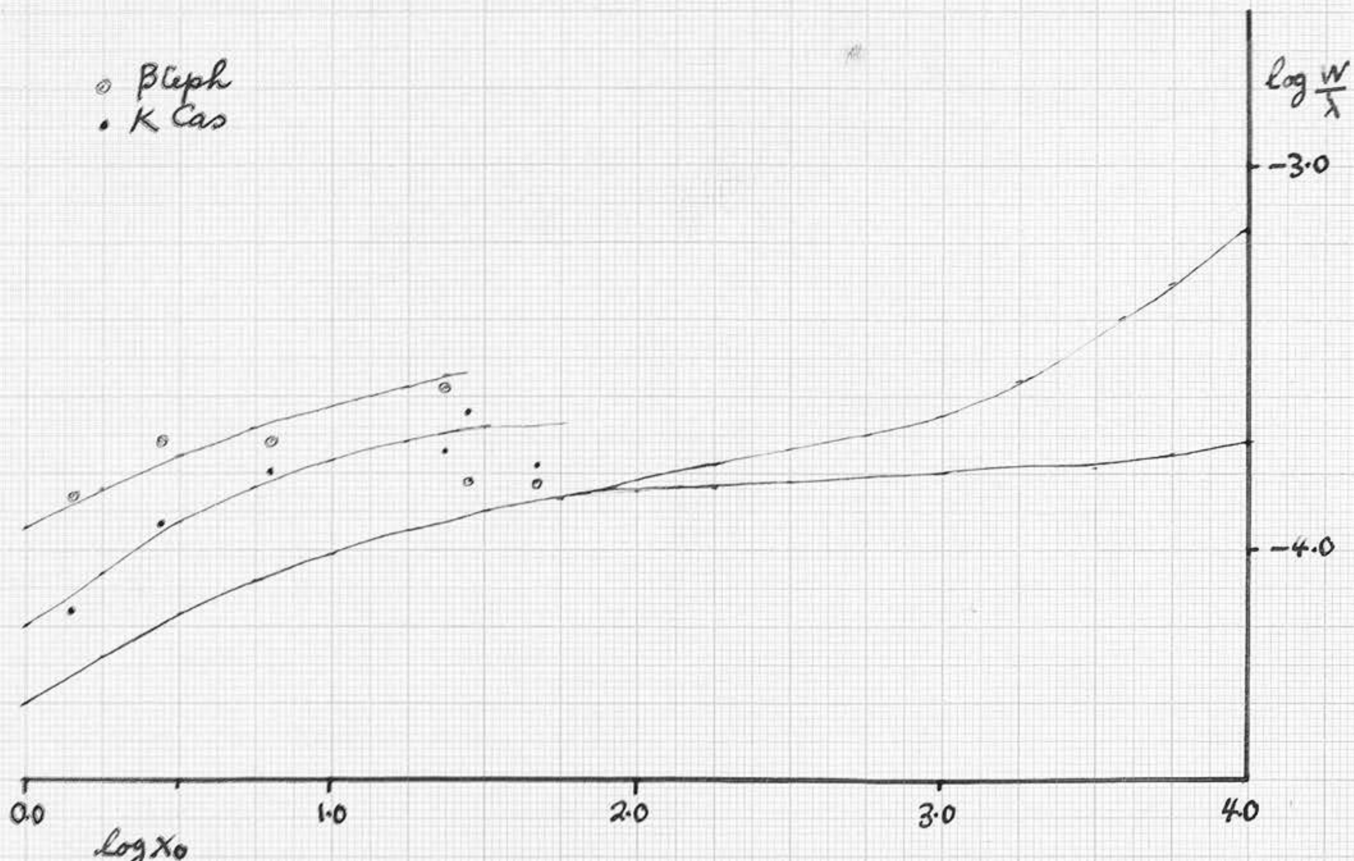

Fig 2. The Theoretical Curve of Growth of Helium (Lower Curve)

The points represent observations in β Ceph & K Cas.

The introduction of V, raises the curve a distance V along the W/λ axis.

(Xo) will also be affected, since it contains ν_0

and $\log X_0 = \Delta' - V + \log X_0' = \Delta + \log X_0'$ where $\Delta = \Delta' - V$

If the observed widths are plotted against (Xo'), the resulting curve will coincide with the theoretical curve except for a vertical shift V and a horizontal one Δ. Thus V and Δ can be obtained from the observations.

Table In the following the measured values of log W/λ and the corresponding log X_0' are listed. X_0' for 6678 and 5876 were computed from Goldberg's paper (27); all the others are taken directly from his table (26). Mean values of W are used.

TABLE X.

Helium Lines in K Cas.

Line	Wave-length	Log X_0'	Log W/λ
$2^3P^o - 3^3D$	5876	1.68	-3.79
$- 4^3D$	4472	1.38	-3.73
$2^1P^o - 3^1D$	6678	1.44	-3.64
$- 4^1D$	4922	.79	-3.80
$- 5^1D$	4388	.45	-3.95
$- 6^1D$	4144	.15	-4.11

The six lines are plotted in Fig.2 against log X_0'. The scatter of the points is reasonable, the mean deviation being .04 along the ordinates.

The mean value of V is + .25, the value of Δ, the horizontal shift is very difficult to determine; a very uncertain estimate is $\Delta = +.10$.

If we assume the difference between [redacted] v and v_0 is due to a turbulent velocity v_t then $v_t^2 = v^2 - v_0^2 = v_0^2 (10^{2V} - 1)$ taking $V = .25$, then $v_t = 1.47\ v_0$

For helium at 20,000 v_0 = 9.1 Km./Sec. and therefore v_t (for κ Cas.) = 13.38 Km./Sec. This is less than the mean turbulent velocity for super-giants of class Bo, as given by Goldberg (26 - Fig.VI - p.637).

It would be very interesting to show that Be-stars, on the average, have less turbulent velocities than super-giants. The present programme at the Royal Observatory, Edinburgh, will help to settle this question, which has an important bearing on the theory of Be-stars.

The total number of atoms in the two levels 2^1P and 2^3P can be obtained from the observed quantities V and Δ by the method of Goldberg (26).

$$\Delta = A' - V = -13.31 + \log \frac{N_a'}{b(T)} - \frac{5040}{T} \chi_J'$$

The number of helium atoms in those two levels are given in terms of the total number of helium atoms by the Boltzmann formula:- $\log N = \log (2S+1)(2L+1) + \log \frac{N_a'}{b(T)} - \frac{5040}{T} \chi_J'$

$(2S+1)(2L+1)$ being the statistical weight of the lower term. Substituting for Δ, V, S and L, we get

$$\log N(2^1P) = 14.14$$
$$\log N(2^3P) = 14.61$$

This is slightly more than the average for Bo.

It should be noted that all the results obtained are subject to the assumptions and methods of Goldberg. Turbulence, as an explanation for the distortion in the curve of growth, is not yet universally accepted (30). Goldberg's allowance for the change of opacity with wave-length, is doubtful. Struve &

Roach (31) suspect some difference between the curves of growth of singlet and triplet series of helium in P Cygni. The observations available are neither numerous nor accurate enough to reveal such an effect in κ Cas.

β Cephei.

During the study, three plates of the B1 star β Cephei were measured. The equivalent widths of the helium lines are given in the next table.

TABLE XI.

Hydrogen & Helium Lines in β Cephei.

Line	1.	2.	3.	Mean	$\log W/\lambda$	$\log X_0'$	Williams
Hβ	2.99	2.91	3.36	3.09	- -	- -	3.44
Hγ	3.85	3.34	3.04	3.41	- -	- -	4.23
6678	.77	.82	1.38	.99	-3.82	1.44	- -
5876	1.21	.62	.80	.87	-3.83	1.68	- -
4922	1.16	.86	.80	.94	-3.72	.79	1.03
4472	1.13	1.30	1.10	1.18	-3.58	1.38	1.38
4388	.75	.77	1.04	.85	-3.71	.45	.74
4144	.57	.56	.60	.58	-3.86	.15	.68
Hδ	3.43	3.52	3.39	3.45	- -	- -	3.98

In this table 1 refers to plate 45-38 taken on 1938, Aug. 26;
2 " " " 51-39 " " 1939, Dec. 4;
& 3 " " " 36-39 " " 1939, Nov. 9.

The units are in A as usual. The separate intensities from the three plates are fairly consistent, except for λ 6678 and 5876.

On plotting log W/λ against log X_0' (Fig.2) we notice that the scatter of the points is big and that the two lines 6678 and 5876 are very much weaker than expected. No conclusion could be reached, since the measures of these lines are bad. But excluding these two lines, a curve may be drawn through the remaining four points. The value of V (vertical shift) comes out to be .44 along the log W/λ axis. The corresponding value obtained by Goldberg from Williams' measures (in the last column) is .52.

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PART IV.

Total Intensities of a Triplet of SiIII in stellar spectra.

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Abstract.

Total intensities of the three Si III lines, 4552, 4567 and 4574 were measured on 81 plates of 32-early type stars. Deviations from the square roots of the multiplet intensities are small, but nevertheless real. Previous measures are included in a discussion of the differences between high and low luminosity stars and between wide and narrow line stars.

The formation of the lines, mainly through damping, is examined. A definite discrepancy is observed between observation and theory. This discrepancy may be removed by assuming a negative turbulence velocity in the atmosphere, an incipient emission filling the lines, or a damping constant more than 440 times the classical value. This last assumption may be nearest the truth.

Total Intensities of a Triplet of SiIII in Stellar Spectra.

Doubly ionised silicon atoms are represented in early B-type stars by the three lines at 4552, 4567 and 4574. They arise from transitions between a single 2S - level and three close 3^3 P levels. The excitation potential of the lower level is 18.92 E.V., and those of the three upper levels are 21.63, 21.62 and 21.61 E.V. respectively. The inner quantum numbers of the four levels are 1, 2, 1 and 0 respectively (1).

These lines appear at O_9 - Bo, reach maximum intensity between B1 and B2 and are hardly seen after B3. A. Fowler (2) considered them a principle triplet, but it might be expected that principal lines would show a more persistent maximum. (4)

As regards intensity, these three lines are very suitable for testing the current physical theories by comparing them with observations. Silicon has four ionisation stages in stellar spectra. Only the neutral atom lines (strongest line λ 3905) are absent in B-type stars. The majority of B-type spectra show two stages of ionisation, and in some stars even three stages are represented. This provides a chance of examining the generalised theories of ionisation. (3)

Relative intensities of the lines provide another chance of testing the theories of line-formation. The three lines in question have the same lower level and very near upper levels; and thus any difference due to exciting energies or wave-length may be neglected.

The simple sum rules of Burger and Dorgelo ~~(#)~~ predict the following ratios for the three lines in emission.

4552 : 4567 : 4574 :: 5 : 3 : 1.

The observed ratios, both in stars and in the laboratory, differ from the above. Struve and Elvey (5) measured the intensities of the lines in the spark between fused silicon terminals. The observed ratios were

4.0 : 2.4 : 1.

The agreement between the ratios and the theoretical ones is not altogether good. The deviation may be real, but the small number of plates used by Struve and Elvey (namely 4) does not permit any definite conclusion.

Relative intensities of the lines in stars were determined in four series of measures. The first one was by Struve and Elvey (5). The investigation was made principally to find whether the relative intensities in stars having nebulous lines are the same as those observed in stars having narrow lines. They have shown that under certain assumptions the ratios of the total absorptions of the three lines in stellar spectra depend upon the form of the absorption coefficient. If the latter is caused by radiation damping, then the total absorptions are proportional to the square root of the theoretical laboratory intensities. If thermal agitation or any other cause produces a flat but extended absorption coefficient, then the total absorptions are directly proportional to the laboratory intensities. They found that, on the average, the square root law is obeyed;

and they did not find any difference between sharp-line and wide-line stars. From this they concluded that the lines are produced by radiation damping, and that wide lines are broadened by rotation and not because of any peculiar absorption coefficient. This agreement with the square root ratios means that if the observed intensities are plotted logarithmically against the laboratory values, the resulting curve would be a straight line of a slope = $\frac{1}{2}$.

During the following two or three years many multiplets (mainly of metals) were observed in the sun and stars, and many deviations from the square root law were reported. The theory of the curves of growth was first developed by Minnaert and Slob (6), who discussed the manner in which the equivalent width of an absorption line varies as a function of the "number" of absorbing atoms. A complete elucidation of the problems of the curve of growth (7) falls naturally into three stages :-

(a): A theory of the formation of absorption lines under the conditions prevailing in stellar atmospheres is needed. All the factors affecting the total intensity of the line should be taken into consideration. So far, only the effect on the scattering coefficient due to thermal motion, and the effect of radiation damping, have been principally discussed. Other factors, such as collisions, stark effect, and hyperfine structure, are considered separately later. Considerable doubt still exists about many basic ideas (such as thermodynamic equilibrium) and constants (e.g. relative multiplet and supermultiplet intensities, oscillator strengths and damping constants.

(b): The theoretically obtained functional relation should be verified experimentally as far as possible. This is a necessary and difficult problem; the available results rest primarily upon measurements of resonance lines.

(c): After verifying our theoretical relation, we can safely apply it to solar and stellar spectra. We have to observe as many lines as possible, and then trace any deviation from a "model" curve of growth back to the physical theories assumed or to the numerical constants used.

This ideal scheme is, needless to say, not complete. Theory and observation have to fill the gaps. The data available for the sun is fairly large and satisfactory. For the stars, especially early type stars, the data is very fragmentary, and sometimes, contradicting. Different curves of growth for the same star (α Cyg.) are obtained by different observers (Pannekoek and Struve), and even the same data in the hands of different workers does not lead to the same result. (8).

In early-type stars, relatively few lines are found, and still fewer of these lines can be grouped in multiplets of known relative strengths. Struve and his collaborators at Yerkes (9) studied the "gradients" of oxygen, iron and titanium multiplets in A and B-type stars. The observed anomalies were explained by large scale "turbulence" in the atmosphere. What concerns us more here, is the difference in gradient between high and low excitation OII lines in B-stars (10), which was at first assumed to be an absolute magnitude effect. After a more careful study this dependence on absolute magnitude was not confirmed. (11)

SiIII lines were re-examined by Elvey (12). The main point was to find any difference between giant and dwarf stars. (By giant and dwarf stars, is meant the luminosity groups given by Struve (3). This division is admittedly uncertain; it depends on the appearance of Balmer-line wings, the presence of forbidden helium lines and on the intensity of interstellar CaII lines. Almost all subsequent investigators have adopted Struve's system, and therefore it will be accepted here, with its limitations.) From his measures Elvey could not find any systematic difference between giants and dwarfs as regards obeying the square root relation. Deviations were observed, but they appeared to be a matter of individual property of the star. This study of Elvey formed the second series of measures of SiIII lines.

The third and fourth series were made by E. G. Williams (13) and by Rudnick (14), almost simultaneously. In these two papers the authors measured the equivalent widths of the strongest lines in B-type stars, but they did not discuss the relative intensities of the SiIII triplet lines. This will be done in the present work.

Summarising, there are three main questions to discuss :-

(1) How far the relative intensities of the SiIII lines, as observed in stellar spectra, agree with the square root relation?

(2) Are the relative intensities the same for giant and dwarf stars?

(3) Are the relative intensities the same for wide-line and sharp-line stars?

The first and last question were answered by Struve & Elvey (5) from twelve measures; while the second was discussed, but not definitely settled by Elvey (12) using twenty-six measures. Errors in measuring faint absorption lines are known to be very large, and the only method to cut them down is by taking the mean of as many observations as possible. The present study is an attempt to observe the relative intensities of the three lines of doubly ionised silicon, and to answer the above questions.

Material and Observation.

The material consists of 81 spectra of 32 stars, photographed on 48 plates. They form a part of the programme of measuring total line intensities of early type stars which is in progress at the Royal Observatory, Edinburgh. All the plates were secured with the II prism spectrograph, attached to the 36" reflector. The region from $H\alpha$ to $H\gamma$ is in good focus. The linear dispersion is 36 A/mm. at 4567A.

The plates were calibrated with a multiple slit calibration spectrograph, the relative slit apertures being determined photometrically by the half-aperture method. Two sets of calibration spectra are obtained on each plate. The plates, Ilford Astra VIII, panchromatic emulsion, were brushed during developing in M.Q. developer at 65°C. for four minutes.

The measures were done on a Moll pattern microphotometer using a thermo couple as the sensitive element. The galvanometer deflections were not recorded, but were read directly on a ground glass scale. The scale is graduated so as to give readings of the quantity Δ, where is a function of the density D, given by the relation

$$\Delta = \log_{10} (10^D - 1).$$

The reduction method used proceeds on the usual lines. Briefly, from the calibration spectra, the rate of change of Δ with the intensity of light is obtained, for each wave-length. This is used in converting the measured in the continuous and in the line into light intensities, and then the ratio of the light intensity inside the line to that outside it, is found. In the present measures, this ratio at the line centre was usually about 90%; it was never less than 80%.

The spectrum of the star was measured throughout a region extending from 4500A to 4625A. Readings were obtained at an interval of .025 mm., except near the lines, where the interval was .010 mm. A density curve was then plotted, from which the continuous was determined.

For plates taken after Feb.6th 1939, neutral filters were used to cut down the intensity of the calibrating light, so as to produce densities comparable with those in the star's spectrum, with equal exposure time. For plates taken before that date a special correction was introduced to allow for the difference in exposure time. (15)

After reducing the density values to light ratios, the total intensity of the line was obtained by integration.

Results and Discussion.

Results are given in Tables I and II. The stars measured are listed in Table I, together with their coordinates, spectral type, luminosity group, line class and number of spectra for each. The actual measures are given in Table II, where the first column gives the series number of the star as listed in Table I, and the second gives the date. The last three columns give the relative intensities of the three lines 4552, 4567 and 4574, taking their total to be 100. The actual total of the three lines is given in column 3, in a unit = .01A of the continuous between the lines. Thus the measured intensity of any line can be obtained by multiplying the sum given in column 3 by its relative intensity.

The list includes all Bo, B1 and B2 stars which have measureable spectrograms available, as well as a few stars outside this range. Harvard classification is used throughout the work.

Errors: The principal aim is to obtain accurate measures, and therefore a careful discussion of the errors involved is necessary. All measures are limited by the instrumental effects, which should be the same for all plates, and thus do not affect differential measures. We may conveniently discuss the following sources of error :-

(1) errors in measurement and reduction;
(2) scattered light;
(3) blending.

(1): Concerning (1), it is noted that all the plates were developed under similar conditions. Owing to the relatively small difference between the intensities inside and outside the lines, the photographic effects are reduced to a minimum, and the method of reduction should be accurate enough. Usually, determination of the continuous spectrum introduces large errors,

TABLE I - LIST OF STARS.

No.	1.	2.	3.	4.	5.	6.	7.
		h. m.	° ′				
1.	γ Peg.	0 8	14 38	B2	4	0	6
2.	K Cas.	0 27	62 23	Boe	i	3	5
3.	ζ Cas.	0 31	53 21	B2	d	0	3
4.	ξ Cas.	0 36	49 58	B3	-	-	1
5.	O Cas.	0 39	47 44	B2e	-	n	1
6.	δ Ceti	2 34	-0 6	B2		0	4
7.	35 Aries	2 38	27 17	B3	-	-	1
8.	40 Per.	3 36	33 39	B2	i	4	1
9.	O Per.	3 38	31 58	B1	-	-	3
10.	ζ Per.	3 48	31 35	B1	g	3	8
11.	ε Per.	3 51	39 43	B3	d	0	6
12.	λ Tauri	3 55	12 12	B3	d	-	1
13.	9 Cam.	4 44	66 10	Boe	g	4	1
14.	ζ Aur.	5 00	41 6	B3	d	7	3
15.	γ Ori.	5 20	6 16	B2	d	3	1
16.	λ Ori.	5 30	9 52	Ge5	g	2	1
17.	ζ Ori.	5 36	-2 0	Bo	d	n	1
18.	ρ Leo	10 28	9 49	Bo	g	3	1
19.	i Herc.	17 37	46 4	B3	i	0	2
20.	102 Herc.	18 5	20 48	B3	d	2	1
21.	ζ Lyr.	19 10	38 58	B3	i	1	1
22.	b2 Cyg.	20 6	36 33	B2e	-	n	1
23.	P Cyg.	20 14	37 43	B1q	-	-	3
24.	55 Cyg.	20 46	45 46	B2	g	1	8
25.	υ Cyg.	21 14	34 29	B3e	-	n	3
26.	β Ceph.	21 27	70 7	B1	d	o	4
27.	9 Ceph.	21 35	61 38	B2	g	2	3
28.	Me Cyg.	21 43	48 51	B3	i	3	1
29.	6 Lac.	22 26	42 37	B3	i	4	1
30.	10 Lac.	22 35	38 32	Oe5	d	o	1
31.	1 Cas.	23 2	58 53	B1	i	o	2
32.	σ Cas	23 54	55 12	B2	-	-	2

1 = Name of star.
2 = R.A. for 1900.
3 = Declination for 1900.
4 = Harvard spectral type.
5 = Luminosity group
g = giants
i = intermediate
d = dwarfs
taken from Struve's list (Ap.J.74, 223, 1931) and from Rudnick (Ap.J.83, 439, 1936)
6 = Line width; arabic numbers denote value of rotation from Struve's list (loc. cit).
n = nebulous-line star.
7 = the number of plates measured in the present investigation.

TABLE II.

Measured Equivalent Widths of Si III lines.

Column (1) gives the serial number of the star (Refer to Table I).

" (2) gives date of observation.

" (3) gives the sum of the equivalent widths of the three measured lines. The unit = .01 equivalent angstroms.

" (4), (5), (6) give the relative intensities of the three lines 4552, 4567, 4574 respectively. The measured intensity of a line can be obtained by multiplying its relative intensity by the sum of the three intensities, given in Column 3.

1.	2.	3.	4.	5.	6.
1	1938-11-30	52	44	33	23
1	1938-12- 3	50	50	28	22
1	1938-12-14	59	59	37	4
1	1939-11-23	79	42	34	24
1	1939-12- 4	58	36	36	28
1	1939-11-23	52	50	25	25
2	1939-11-23	116	47	38	15
2	1939-11-15	147	43	37	20
2	1938-10-25	121	42	40	18
2	1938-11-15	142	37	37	26
2	1939- 1- 9	125	36	40	24
3	1939-11-15	37	65	11	24
3	1939-12- 5	55	34	33	33
3	1939-11-24	50	44	40	16
4	1939- 1- 5	19	37	42	21
5	1938-12-14	33	36	28	36
6	1940- 1-22	70	37	30	33
6	1938-12- 3	43	56	26	18
6	1938-11-19	78	50	37	13
6	1938-12-14	62	52	27	21
7	1940- 1- 4	21	48	31	21
8	1940- 1- 4	37	49	34	17
9	1940- 1-17	181	48	30	22
9	1940- 1-13	114	43	31	26
9	1938-11-15	74	44	28	28
10	1938-11-19	184	40	44	16
10	1938-12-14	148	42	35	23
10	1939- 1- 4	98	44	28	28
10	1939- 1-11	110	42	33	25
10	1940- 3- 1	81	42	35	23
10	1940- 3- 5	111	43	40	17
10	1940- 3- 6	113	41	36	23
10	1940- 3-25	130	50	28	22

TABLE II - (Contd.)

1.	2.	3.	4.	5.	6.
11	1940- 3- 5	74	44	38	18
11	1940- 3- 6	104	40	35	25
11	1940- 3-25	52	46	35	19
11	1938-12-14	89	41	34	25
11	1939- 1- 4	100	47	36	17
11	1940- 3- 1	71	47	32	21
12	1940- 1-13	31	45	32	23
13	1939- 1- 5	24	50	21	29
14	1940- 3- 5	43	28	42	30
14	1940- 3-25	24	50	21	29
14	1940- 3- 5	37	38	36	26
15	1939-12- 4	76	55	26	19
16	1940- 1-13	26	46	27	27
17	1939- 2- 7	30	47	33	20
18	1940- 4- 4	134	45	31	24
19	1939- 9- 5	50	66	20	14
19	1939- 8-18	13	31	38	31
20	1939- 7-24	32	62	13	25
21	1939- 9- 5	39	38	41	21
22	1939-10-19	20	40	40	20
23	1938- 9- 9	147	45	38	17
23	1938- 9-20	155	51	26	23
23	1938- 8-22	103	39	36	25
24	1938- 8-26	91	43	36	21
24	1938- 9- 9	117	45	34	21
24	1938- 9-20	116	46	33	31
24	1940- 9-17	123	40	27	33
24	1940- 9-21	91	36	43	21
24	1938- 9-21	99	52	32	16
24	1940-10-11	84	42	29	29
24	1940-10-12	68	49	34	17
25	1939-11-23	45	53	31	16
25	1938-10-25	16	62	25	13
25	1939-11-24	45	40	31	29
26	1939-11- 9	76	45	38	17
26	1938- 9-25	94	40	44	16
26	1938- 8-26	90	40	30	30
26	1940- 9-17	108	40	28	32
27	1939-11- 9	90	41	29	30
27	1939-10-19	70	46	31	22
27	1938- 9- 9	100	53	35	12
28	1939-10-19	58	41	45	14
29	1938-11-19	92	51	35	41
30	1938-11-15	35	55	20	25
31	1938-11-22	135	39	34	27
31	1938-11-26	77	35	47	18
32	1938-12-14	34	38	47	15
32	1938-11-19	89	41	34	25

because of this measurements were extended far on both sides of the lines, so as to ensure a determination of the continuous as accurate as possible. However, if we have in view the determination of the intensity ratios of the lines only, then if the background is drawn either too high or too low, we have a considerable change in the measured value of total absorption, while the intensity ratios are affected but little. This result was pointed out by Shajn (16) for the H and K lines, and it should apply equally well to any other lines, provided that the background is drawn parallel to the true continuous. This condition is easily satisfied in our case, since the lines are within 25A from each other.

The slit width used in the photometer varied from .010 mm. → .022 mm; it is generally known that no serious error can result from variable photometer slit-width.

The linear dispersion on the plates was determined carefully from two spectra of iron arc, and from the separation of the lines themselves in some sharp-line stars. The adopted dispersion is the mean of many such determinations. Almost all the sources of error mentioned above will affect the three lines in the same way, and it may be safely assumed that the relative intensities of the lines are more accurate than the actual equivalent widths, which may vary from plate to plate.

(2): Scattered light tends to decrease the measured intensity, since it fills the line to a certain extent. [redacted]

A test was made with the object of investigating whether the total intensities depended on exposure.

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The measured total intensities for ξ Per. and 55 Cyg. were plotted against Δ, eight measures being available for each star. No systematic tendency could be seen. A comparison between the total intensities measured at Edinburgh and those published by other observatories, shows a tendency towards higher intensities for Edinburgh measures. This difference is large for faint lines and almost disappears for strong ones.

(3): From the lists given by Struve (3), by Marshall (17), in their study of B-type spectra, it is found that 4552 is the only line seriously blended. The blending lines are due to NII and SII. By comparison with other unblended lines of NII and SII it is found that the error in 4552 should not be more than 1 or 2 in the visual scale of intensity used by those writers. The effect of blending will increase the measured total intensity of 4552, and as a result its relative intensity will be increased by a quantity d, and the relative intensities of the two other lines will be decreased by two quantities totalling d. Thus if the true relative intensities of the three lines are

$$A : B : C \quad \text{so that} \quad A + B + C = 100$$

and due to blending the measured values will be proportional to

$$A + d : B : C = 100 + d,$$

then the calculated relative intensities will be

$$A + d\,(1 - \frac{d}{100}) \ : \ B\,(1 - \frac{d}{100}) \ : \ C\,(1 - \frac{d}{100}),$$

deviating from the true ratios by values proportional to

$$+ (100 - A) \ : \ - B \ : - C.$$

Taking A = 45, B = 35, and C = 20 (these are square root ratios) then the deviations of the observed relative intensities due to the blending of 4552 (if it were effective), should be proportional to

$$+55 : -35 : -20.$$

The observed deviations (from Table III) are in the ratios

$$-3 : -17 : +20.$$

Thus the blending of 4552 cannot produce the observed deviation; on the contrary, it seems that such blending would tend to diminish the deviations.

The probable error of a single plate: Complete sets of measures are available for ξ Per. and 55 Cyg. Probable errors were calculated for the means of total absorption (Column 3 - Table II) and the three relative intensities. The results are as follows :-

Star	No. of Measures	Mean Total Absorption (.01A - Unit)	Relative Intensities % 4552	4567	4574
55 Cyg	8	98.6 ± 4.9	43.9 ± 1.2	33.5 ± 1.0	23.6 ± 1.8
ξ Per	8	122.0 ± 7.7	42.9 ± .7	34.6 ± 1.3	22.5 ± 1.0

The corresponding probable errors for a single plate are obtained by multiplying the P.E. of the means by 2.83 ($\sqrt{8}$).

The mean P.E. of the relative intensities in a single plate is ±3.4, which is of the order of 10%.

The Relative Intensities.

It has been shown that the relative intensities are measured more accurately than the total intensities since they are less susceptible to systematic errors.

The different intensity ratios obtained by previous observers are listed in Table III. together with the number of measures in each case. The ratios are predicted by the sum rules and their square roots are also shown in the table. The ratios of Williams and Rudnick were computed from their published results (13, 14). The probable errors for the mean of Edinburgh measures were obtained using the sum of residuals and not the squares of the residuals. For such a large number of observations the two formulae lead to practically identical results.

Relative Intensities - TABLE III.

Observer	4552	4567	4574	No. of observations
- - multiplet ratios	55.6	33.3	11.1	- -
Square root of multiplet ratios	45.0	34.9	20.1	- -
Observed in emission spark	53.1	32.4	13.5	4
Stellar ratios by:- Struve and Elvey	46.7	31.1	22.2	12
Elvey	41.0	34.4	24.6	26
E.G. Williams	46.4	35.2	18.7	27
Rudnick	45.7	32.8	21.5	34
New observations	44.7 ± .3	32.8 ± .3	22.5 ± .2	81
Mean of all stellar ratios (without Edinburgh)	44.7	33.5	21.8	99
Final mean	44.7	33.2	22.1	180
Edinburgh deviations from $\sqrt{\ }$ law	-.3	-2.1	+2.4	
Deviations of final mean	-.3	-1.7	+2.0	

The deviation for 4552 is of the same order as the probable error, and is therefore ~~is~~ not significant, but for the other two lines the deviation is considered real. It cannot be due to any blending of the line 4552 alone. The observed ratios agree with a hypothetical blending of 4552 and 4574 by +2.4% and +3.6% respectively. There is no physical reason to justify such a blending.

Another factor which might explain the deviations is the non-homogenity of the observations. The ratios may change with spectral type, luminosity class, line width or/and intensity of the lines. These points will now be discussed.

Difference in intensity ratios between high and low luminosity stars.

The stars were grouped according to Struve into three luminosity classes. The results are given in the next table.

TABLE IV.

Observer			4552	4567	4574
Struve & Elvey - - - - - -	g	(1)	41.0	33.4	25.6
	i	(5)	46.2	32.8	21.0
	d	(6)	46.9	30.5	22.5
Elvey - - - - - - - - - - -	g	(14)	41.3	35.2	23.5
	d	(12)	40.6	33.6	25.8
Williams - - - - - - - - -	g	(13)	45.5	35.8	19.5
	i	(2)	47.4	34.5	18.1
	d	(12)	47.0	35.5	17.5
Rudnick - - - - - - - - -	g	(11)	44.4	34.4	21.2
	d	(23)	46.6	31.8	21.6
Edinburgh - - - - - - - - -	g	(27)	43.8	33.8	22.4
	i	(8)	43.8	36.7	19.5
	d	(31)	46.1	31.0	21.9

The difference between giants and dwarfs in relative intensities is best seen when they are referred to their mean, and not to the theoretical square root ratios. The differences thus obtained from every series of measures are completely independent.

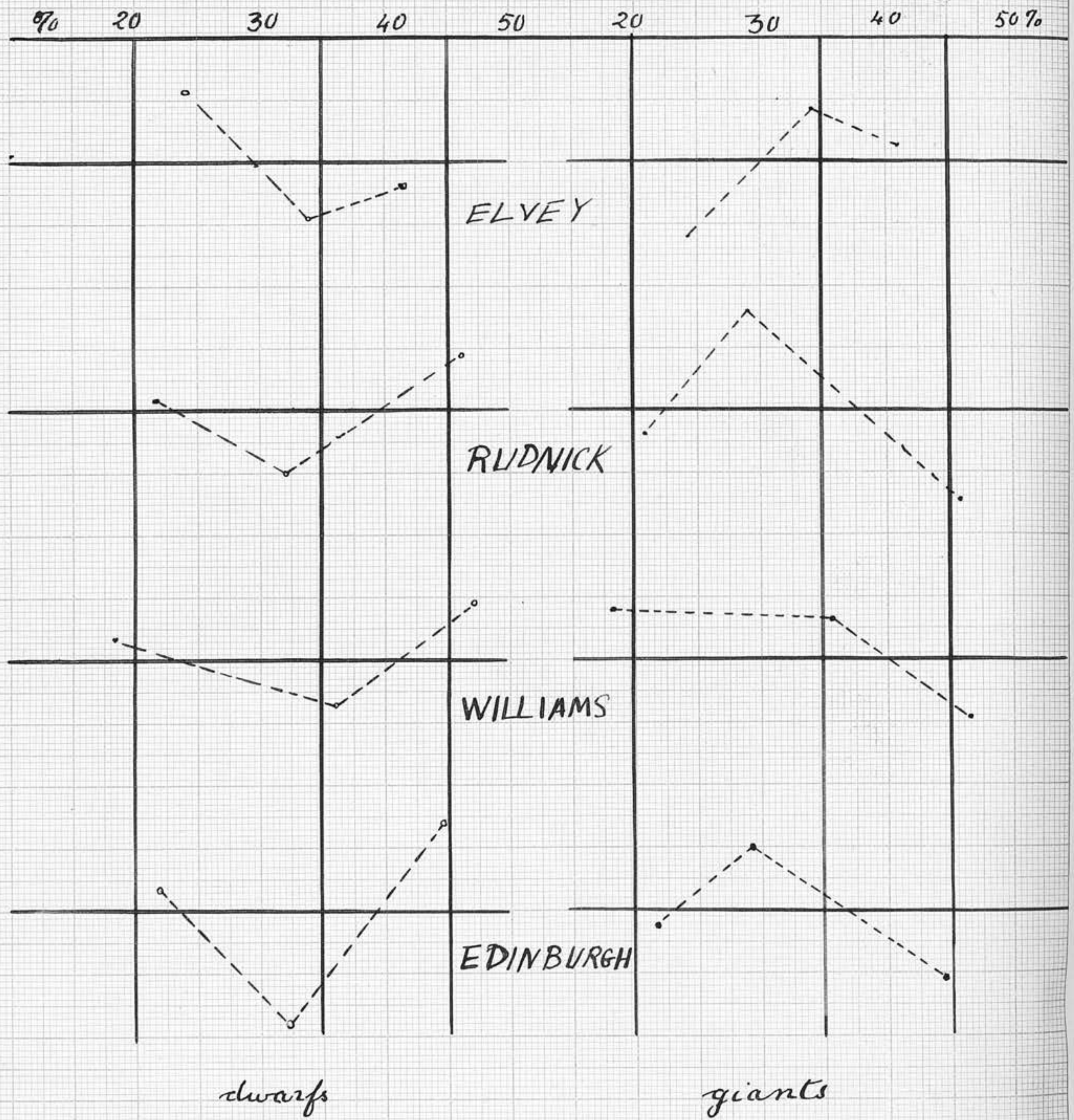


Fig I

Deviations of the relative intensities of Si III lines for giant and dwarf groups, from the mean relative intensities. Abscissa mean relative intensities [total = 100] of the 3 lines 4552-4567-4576-from right to left. Ordinates:- deviations from the mean. [scale = 5 times the scale abscissa

The numbers of measures in each group are not equal for all observers. Any error due to this unequal distribution of weights will be neglected. The results are shown graphically in Fig.I. The scale of deviations (ordinates) is taken five times larger than the scale of the relative intensities (abscissa). The three vertical lines correspond to the relative intensities given by the square root of theoretical ratios (1 : 3 : 5).

4567 (the middle line) is always stronger than the average in giants, and weaker in dwarfs. The strengthening of strong OII lines in giants, and their weakening in dwarfs was discovered by Struve (10). As can be seen from Fig.I, this is not the case for Si III. There is no explanation available for the systematic strengthening of 4567 in giants, and the opposite effect in dwarfs. It is unlikely that the agreement between all results is purely accidental.

Variation of relative intensities with spectral type.

Observations by Struve and Elvey, and by Elvey, are not numerous enough to be included in this section. Rudnick's and Williams' data, as well as the writer's, were grouped according to spectral types Oe5, Bo, B1, B2 and B3. The refined classification used by Williams was kept for his observations. The resulting deviations of the lines in each subdivision were plotted in the same way as in Fig.I, and similar figures were obtained, some of them resembling the curves of giants in Fig.I and others resembling the curves for dwarfs. This was found to depend largely on the number of measures of each group used in deriving the mean ratios for each sub-division. For example, 13 spectra of class B1 were measured in

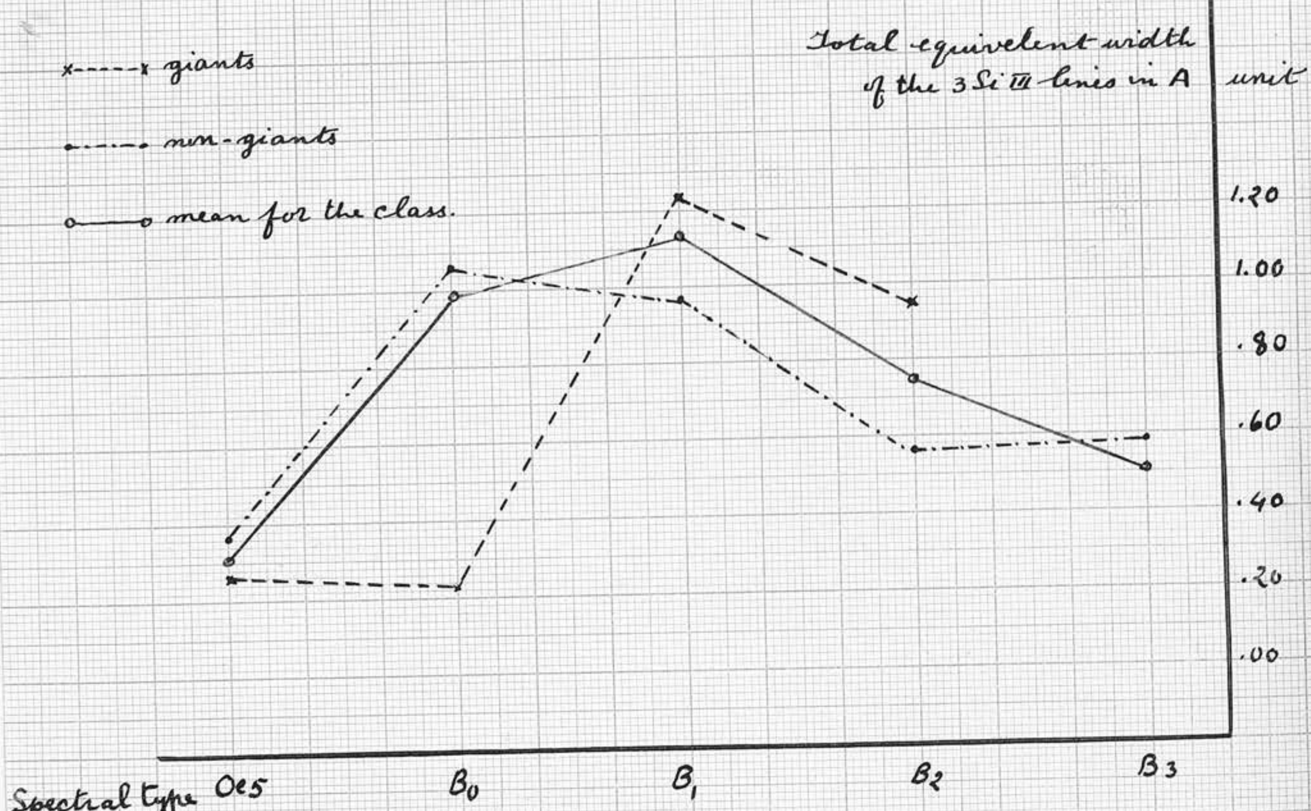
Edinburgh, 9 of which are giants and 4 dwarfs. The mean of the 13 (representing the sub-division B1) deviates from the mean of all measures, exactly in the same way as the mean of all giants (See preceding section and Fig.I) does from the mean of all measures. This correspondence was found to hold for every sub-division and for the three series of observations (Williams, Rudnick and Edinburgh) discussed. Thus the change of intensity ratios with spectral type cannot be detected by this method. Incidentally, the result confirms the difference between giant and dwarf stars shown in Fig.I.

<u>Sharp-line and wide-line stars</u>:- No systematic difference could be detected between these two groups of stars. This is in agreement with the previous observations by Struve and Elvey (5).

<u>The total intensities</u>:- The maximum strength of Si III - lines is usually considered to be between B1 and B2. Giants have stronger lines than dwarfs. This rule seems to be always without exception. Theoretical curves for the intensities of Si III were given by Milne (18) for the values of surface of gravity $g = 10^3, 10^4$. The curve of smaller g reaches maximum before the other, and the two curves approach each other gradually, and coincide on the descending branch, giving thus Milne's null-effect.

Mean total intensities of the three lines for each subtype, obtained from the present data, are given in the next table.

Fig II



Variation of the intensity of Si III with spectral type

TABLE V.

Sub-division	Giant stars		d & intermediate stars		Mean for sub-division	
Oe5	26	(1)	35	(1)	31	(2)
Bo	24	(1)	107	(6)	98	(7)
B1	123	(9)	97	(6)	114	(18)
B2	95	(11)	57	(15)	75	(31)
B3	- -	- -	59	(15)	50	(21)

The only giant of class Bo is 9 Cam, which was classified O9 at Victoria and by Williams, and its position in Fig.II. places it earlier than Bo. The curve in Fig.II is very similar to Miss Payne's (19) and Williams (13) in having a sharp maximum compared with the flat maximum given by Rudnick (14). Giants have stronger lines in spectral types later than Bo, but for Bo and Oe5 the present material (9 plates) indicates that this is not the case.

The Formation of the Lines.

The formation of the lines: - It has been shown that the observed relative intensities of the silicon lines agree, except for a small deviation, with the relative square root ratios of the multiplet intensities obtained from the simple sum rule. The following values are taken from Table I.:-

Line	$\log \dfrac{\text{Observed relative intensity}}{\text{Relative intensity from the square root laws}}$
4552	.00
4567	-.02
4574	+.04

The final observed ratios are the mean of 180 measures (81 in Edinburgh) by 5 different observers, using different instruments. The deviations are more than the probable errors and must be considered real.

The agreement with the square root ratios indicates clearly that the lines are formed mainly by damping, and that the Doppler absorption core can be relatively neglected. The position of the lines on the curve of growth must be on the damping branch with inclination $\frac{1}{2}$. This agreement seems to hold for all spectral classes discussed, namely from B3 to Oe5, although measures are less certain, and less numerous, for stars with faint lines. The distribution of points on a diagram connecting the relative intensity of 4552 with the total equivalent widths of the three lines in the same star, does not show any tendency in the direction of having higher relative intensities for 4552 in weak-line stars. Such a tendency is expected if the square root relation was not obeyed in weak-line stars. This result is confirmed by studying the scatter of 4574. This line should have its relative intensities smaller in the case of direct proportionality to multiplet intensities than in the case when the square root intensities are followed.

The mean equivalent widths of the three lines range from .31 A in class Oe5 to 1.14 A in class B1. The square root ratio is obeyed all over this range. The equivalent widths of the separate lines in these two extreme cases will be :-

Line	4552	4567	4574
Strongest line (B1)	.510	.378	.252 A
Faintest line (Oe5)	.139	.103	.069 A,
and log equivalent widths will be			
for strong lines	-.29	-.42	-.60
for weak lines	-.86	-.99	-1.16

The theoretical work of Minnaert and Slob (6) will be used in the following discussion. They have computed curves giving the equivalent width of a line as a function of the concentration of the atoms taking part in its production for different values of the parameter $a = \frac{\nu'}{b}$ where ν' is the classical damping constant

$$\nu' = \gamma = \frac{8\pi^2\epsilon^2}{3mc\lambda_0^2} = 1.09 \times 10^8 \quad \text{for} \quad \lambda_0 = 4500\,A$$

and $b = \frac{2\pi}{\lambda_0}\sqrt{\frac{2kT}{m}}$, m being the atomic weight and the other constants have their usual meanings. The quantity (b) is a measure of the Doppler core of the absorption line. The series of curves published by Minnaert and Slob (Fig. I, p. 547) depend only on the one parameter a, and they were computed for λ = 4500A and bo = 1.70×10^{10}. For other wave-lengths and other (b) (which is variable with temperature, element and wave-length), the measured equivalent width has to be multiplied by a factor

$$\frac{1.70 \times 10^{10}}{b} \times \left(\frac{4500}{\lambda}\right)^2$$

The Doppler constants b are given in their paper in Table I for different elements and wave-lengths and for T = 5000°; assuming a mean temperature of 20000°, and neglecting the difference between 4500 A and the mean wave-lengths of the three Si III lines, the value of b = 4.78×10^{10} is obtained.

The application of this factor gives for the log A of the strongest and faintest lines given above the following values

log A for 4552 in B1 (strongest) = - .74
log A for 4574 in Oe5 (faintest) = -1.62

Throughout the range between these extreme values, all the equivalent widths were found to agree very closely with the square root relation. Thus these two values of log A should fall on the damping branch of the appropriate curve of growth, which has an inclination $= \frac{1}{2}$.

This condition is not satisfied by any of the curves given by Minnaert and Slob; the nearest curve to satisfy it is that for which a = 1. Actually a curve with a parameter a > 1 will fit better.

The result is that $\nu' \geq b = 4.78 \times 10^{10}$; this is more than 400 times the classical value.

Many observers have reported values of the damping constant several times the classical value (7), both in the sun and in stars. The largest value reported is of the order of 20 γ where γ is the classical constant.

Minnaert and Slob based their calculation on Voigt's formula of the line scattering coefficient, and took no notice of collisional damping. Pannekoek (20) discussed the influence of collisions on the formation of the Frauenhofer lines. He obtained a value of the damping constant 19 γ in α Cyg, while Minnaert and Mulders (21) obtained a value = 9 γ for the sun. This is rather an unexpected result, since α Cyg. is a supergiant and the sun is a dwarf.

The excessive value derived for the silicon lines is too large to be readily acceptable. It may be plausible to recapitulate the steps by which this value is obtained. The relative intensities of three lines forming a simple multiplet were measured in 180 spectra of early type stars by five different observers, and were found to agree with the square roots of the theoretical multiplet intensities. This observational fact is combined with the theoretical work of Minnaert and Slob, and the result that $a \gtrsim 1$ is obtained.

From this it follows that either

(1) (the damping constant) = b = 4.78×10^{10} = 440 γ

or (2) If we do not accept (1) above, it may be thought that the calculated value of b is larger than the real one. This means that the true Doppler motion is less than that deduced from the assumed temperature. This would result from a strong "negative" turbulent motion, which cannot be physically visualised.

or (3) The discrepancy may not be completely real; it may be simply a reflection of some approximation used by Minnaert and Slob in computing their curves. This may be possible since collisions were not considered by those authors.

In more recent papers by Menzel (22) and by Baker (23) three equations for the curve of growth were obtained in the form

$$\log \frac{W}{\lambda} = F(X_o)$$

where W is the equivalent width of the line in Angstroms, λ its wave-length, Xo the optical depth at the centre of the line, and F is a certain function having three different forms, one for small values of W to be used near the centre of the line, one for large values of W, to be used in the wings and the third for intermediate values of W, and is used for the "transition" stage of the curve of growth.

The damping factor Γ used by Menzel and Baker is defined as the sum of the reciprocal mean life times of the two energy levels involved in producing the line. It includes damping due to radiation and to collision; it enters only in the damping branch of the curve. Γ is known accurately in a few cases. Generally a mean value of Γ of the order of 10^9 is adopted.

Using Menzel's (22) formula, Goldberg (24) calculated a theoretical curve of growth for helium lines in early-type stars. He adopted two values of Γ; the first equals 2×10^7 for the triplet lines whose lower level is metastable, and the second 2×10^9 for the singlet levels. The last value is of the same order as that of the metals.

Before using Goldberg's curve allowance has to be made for the difference between the atomic weight of helium (4) and that of silicon (28). The Doppler and intermediate parts of the curve have to be lowered by $\log \sqrt{28/4} = .42$ along the $\log W/\lambda$ scale, and the damping branch should be lowered half as much. There should be corresponding shifts in the horizontal scale, but these do not concern us here. The damping branch in the modified curve will begin at a point corresponding to $\log W/\lambda = -4.1$ approximately. For the strongest and faintest lines of Si III given above we find $\log W/\lambda = -3.95$ and -4.82 respectively. The straight line connecting the points will not coincide with the damping curve, as the observations demand.

Thus we reach the same conclusion that a larger value of the damping constant or a smaller value of the Doppler core is needed to fit the observations. In the method of Menzel the three separate parts of the curve of growth are joined asymptotically to form the complete curve, and therefore no more definite value of the necessary damping constant can be ascertained.

The value of the damping constant found from Minnaert and Slob's curve is $4.78 \times 10^{10} = 2.39\ (2 \times 10^9)$, that is to say, about 24 times the damping assigned by Goldberg for the singlet lines of helium.

The agreement between the results obtained from the two theoretical curves of growth makes it more probable that the discrepancy between the observations and theory is real.

In many cases (cf. 20) the distribution of intensities within some multiplets was found to be very anomolous, and it is dangerous to derive general conclusions from a few number of lines. Thus it is emphasised that the results given here refer only to the multiplet of Si III. Other multiplets of other elements, or even the same element, may be behaving quite differently.

If collisions were the important cause of the excessive damping, then there would have been a difference between high and low luminosity stars. As far as the available material can show, no such difference is found. To calculate the influence of collisions from physical data is difficult (7), because the radii of collisions have not been determined for single atoms colliding with such slow electrons as mainly occur in stellar atmospheres.

There is a possibility, which may be worth mentioning. The true total intensities of the lines may be larger than the measured ones, because of some incipient emission, which fills the absorption lines. This suggestion may help in explaining the discrepancy, but it is unlikely that it would be sufficient, since a rough estimation shows that the observed total intensity must be only one seventh the real one (without emission). This is rather incomprehensible. Such an effect is expected more in supergiant and emission-line stars than in others. In the

Edinburgh list (Table II) there are 27 measures of such stars. This number is insufficient to settle this question. Giants cannot be directly compared with non-giants, since the latter have usually fainter lines, which may be still fainter than those of giants, even if the latter were affected by emission.

The large value of damping constant, (440 times the classical value), may not be so very uncommon, as one may think. In a recent study of the spectra of supergiants of type M, Lyman Spitzer Jr. (25) found that the variation of the atomic absorption coefficient a_λ, with increasing distance from the line centre λ, shows that a_λ is given by the usual radiation-damping formula with a damping constant some five hundred times the classical value. This conclusion is obtained from a totally different reasoning, but it may indicate that large damping constants are to be expected even in the atmospheres of supergiants.

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